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INTRODUCTION TO HYDRAULIC AND PNEUMATIC DRIVES

Practical works



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Introduction

The proposed laboratory manual supports the training of students taking courses in hydraulic and pneumatic drives, or hydraulic equipment.

Students find, in addition to the theoretical elements specific to each paper, practical and numerical applications related to the physical properties of hydraulic fluids (liquids or gases) used in drive installations.

In the first part, the manual contains eight practical papers that cover a broad range of topics in the field of hydraulic and pneumatic drives. They are laboratory works that can be carried out in the university laboratories, using the facilities and stands provided, and allow students to determine the physical properties of fluids used in industry (fuels, oils, detergents, hydraulic fluids, alcohol, etc.), as well as the possibility of creating drive schemes.

Each paper has a theoretical description of the targeted activity, a presentation of the working method and instructions for carrying out the work.

The second part of the manual contains numerical applications specific to the properties of fluids used in hydraulic installations. We provide a calculation breviary containing the formulas necessary to solve numerical applications.

To exemplify the approach to these types of applications, the laboratory manual contains a chapter where solved problems are presented, specific to the properties of hydraulic fluids and hydraulic and pneumatic drives. In the final part, several numerical applications are proposed for solution.

The laboratory manual aims to support the preparation of students, creating an easy connection between the theoretical and practical knowledge that they acquire during the hydraulic and pneumatic drives course.

Understanding Parameters in Hydraulic and Pneumatic Machines

Hydraulic and pneumatic systems stand as pillars of power transmission, actuation and control. From robotics applications to excavators and presses, these fluid power technologies are encountered in countless areas of the industry. At their core, both hydraulic and pneumatic systems leverage the principles of fluid mechanics to generate and transmit power. Therefore, we find the need to understand and compute their working parameters. This knowledge is essential for achieving peak performance, ensuring safety standards, maximizing energy efficiency, extending system longevity and leading to cost effectiveness. Without this fundamental comprehension, systems can become inefficient, unreliable, transforming powerful tools into costly liabilities.

Defining key parameters

For hydraulic and pneumatic systems it is essential to define the core parameters that govern the behaviour and performance of the machinery. Each parameter plays a unique role, influencing everything from force generation and speed to component wear and energy consumption.

Hydraulic parameters

Hydraulic systems rely typically on incompressible fluids, such as hydraulic oil, and are renowned for their ability to transmit high forces with precision. Their key parameters include:

- **Pressure:** One of the most critical parameters in hydraulics. We can find several factors that define the type of pressures exerted in a hydraulic system: working pressure, burst pressure- maximum pressure on a component and relief valve pressures;

- Flow rate: Given by the volume of fluid passing a point per unit of time. It is influenced by the velocity of the flow, impact speed, being crucial for controlling the overall performance;
- Fluid viscosity: dependent by the type of fluid, working temperature and friction. It can exert a high impact on the system efficiency, lubrication quality and generated heat;
- Temperature: It can lead to knowing the operating range of the system, fluid degradation, shorten the lifespan of seals and rings, leading to leaks, and can have an impact on the fluids viscosity.
- Contamination level: defined by foreign particles in the hydraulic fluid;
- Component sizing: knowing the physical dimensions and capacities of individual components is a key factor to constructing a performant system.

Pneumatic parameters

Pneumatic systems are known for their speed, simplicity and cleanliness, though typically operating at lower forces than hydraulic ones. Their key parameters include:

- Pressure: it is divided into several aspects: supply pressure- given by the compressor, working pressure, regulated pressure;
- Flow rate: Characterized by the volume of air passing through a point in a unit of time. It is influenced by the speed and air consumption.
- Air quality: Moisture can affect pneumatic systems leading to corrosion, as well as lubricating fluids that can contaminate the environment. It has a direct impact on the lifespan of the system;
- Temperature: It has an influence on the overall performance and can have an impact on the seal life;

- Component sizing: The optimum physical dimensions of the system lead to high performance and efficiency;
- Leakage: Has an impact on efficiency, even the small leaks can lead to significant energy waste due to the continuous operation of the compressor that needs to compensate.

Enhancing performance and efficiency

The importance of knowing hydraulic and pneumatic parameters becomes apparent when considering the optimization of system performance and energy efficiency. Precise parameter management allows engineers and technicians to extract maximum utility from these systems while minimizing waste and energy.

There are several factors that lead to increasing the performance of a system. One of them is optimizing the design and sizing. This states the importance of the initial design and accurate sizing of fluid power systems. Understanding the requirements such as speed, force and duty cycle of an application allows the precise selection of the components. For instance, knowing the maximum load that can be lifted by a hydraulic system will influence the operating pressure. Also, if a certain lifting speed is required it will lead to knowing the necessary pump flow rate. In pneumatics, the force required to move a workpiece dictates the cylinder bore, while the cycle time influences the valve's flow capacity and the overall air consumption.

Regarding the optimum design, the engineers have to avoid oversizing and undersizing. Selecting components that are too large or powerful for the desired application can lead to significant energy and material waste. An oversized hydraulic pump will consume more power than necessary, generating excess heat that must be dissipated. An oversized pneumatic compressor will run inefficiently, cycling on and off frequently and

potentially leading to higher costs. Undersizing components will struggle to meet performance demands, leading to sluggish operation, excessive heat generation, pneumatic wear and system failure. An undersized hydraulic cylinder won't generate enough force, while undersizing pneumatic valves will lead to flow restrictions, reducing the actuator speed.

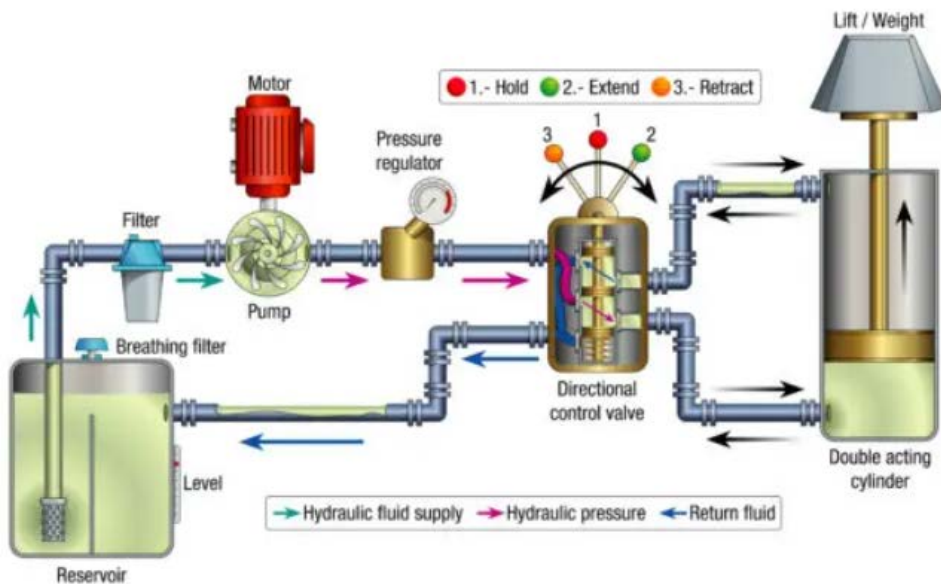


Fig.1. Basic hydraulic system by Pandragon systems[29]

A high influence holds the pressure drop. Excessive ones in hydraulic lines reduce the available power at the actuator and generate unwanted heat. In pneumatic systems, significant pressure drops mean less effective pressure at the point of use, requiring the compressor to work harder and consume more energy to maintain the desired working pressure.

Another factor affecting the performance is energy consumption. Fluid systems are significant energy consumers, and precise parameter control is needed for efficiency. In hydraulic systems, power is directly proportional to pressure and flow. Understanding this relationship allows engineers to optimize the operations of the system. For example if the systems requires a high force only at a certain interval of time, reducing

the pressure during idle times can lead to energy saving. As previously discussed, incorrect fluid viscosity leads to increase internal friction and pressure losses, directly translating to wasted energy. Operating systems outside the optimal temperature range can cause the fluid to become too thick or thin, impacting pump efficiency and increasing energy consumption.

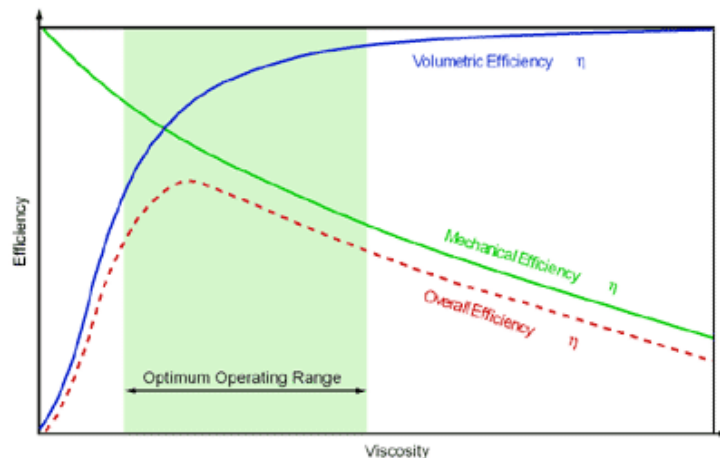


Fig.2. Viscosity to pump efficiency provided by lubricant producers [30]

An aspect to take into account is reducing leaking in pneumatic systems. Compressed air is expensive to be produced. Small leaks can add up to substantial energy losses over time. Knowing typical pressure levels and being able to detect deviations can point to leakage issues, enabling timely repair and significant energy savings.

The ability to precisely control the speed and the force of actuators is an important aspect of the fluid power systems, directly tied to parameter knowledge. In hydraulics, flow control valves are used to regulate the speed of cylinders and motors. Knowing the required speed and the displacement of the actuator allows for accurate calibration of these valves. In pneumatics we use similar flow restrictions. Also, for both hydraulic and pneumatic cylinders, the generated force is a direct product of the operating pressure and the effective area of the piston. Precise pressure regulation ensures that the exact force required for a task is

applied, preventing damage to workpieces or machinery. Understanding the inherent parameters of the system is crucial for predicting and optimizing response times. Proper tuning of control valves and damping mechanisms, based on these parameters, ensures system stability and prevents oscillations or uneven movements.

Optimum parameters lead to system stability and responsiveness. In Hydraulic systems, if the inlet pressure drops too low cavitation can occur, where vapor bubbles form and then collapse, causing significant damage and noise. Air entrainment can also occur. Both issues severely degrade performance and efficiency. Knowledge of fluid properties, flow rates, and pressure drops helps in designing systems that prevent these phenomena. Hydraulic accumulators store energy and dampen pressure pulsations. Correctly sizing an accumulator requires knowledge of system pressure fluctuations, required energy storage, and fluid volume changes. An undersized accumulator will be ineffective, while an oversized one will lead to unnecessary expenses.

In many applications, smooth, controlled motion is required. Understanding the system's dynamic parameters allows for the selection and tuning of damping to achieve the desired motion profile and prevent shock loads. In essence, a deep understanding of these parameters that transform fluid power in movement is a precise science, leading to systems that are not only powerful but also highly efficient, responsive, and tailored to their specific tasks.

Ensuring safety and reliability

Beyond performance and efficiency, the knowledge of hydraulic and pneumatic parameters is absolutely necessary for ensuring the safety of personnel and the long-term reliability of the machinery. Fluid power

systems operate under significant pressures and can pose severe hazards if not properly understood and managed.

Prevention is a key factor to ensuring a safe working environment. Every component in a fluid power system (hoses, pipes, cylinders, pumps, etc.) has a specified burst pressure (the pressure at which it is designed to fail). Operating pressures must always be well below this limit, typically incorporating a safety factor of 2:1, 3:1, or even 4:1 for critical applications. Knowledge of these parameters is crucial during design and operation to prevent catastrophic failures that can lead to explosive ruptures, fluid sprays, and uncontrolled machine movements. Relief valves are critical safety devices designed to protect the system from overpressure. If the system pressure exceeds a set point, the relief valve opens, diverting fluid and preventing damage.

Knowing the maximum safe operating pressure for all components allows for the correct setting of these valves. High temperatures are a silent killer of hydraulic systems. Excessive heat accelerates the oxidation and breakdown of hydraulic fluid, reducing its lubricity and forming corrosive byproducts. This degraded fluid then attacks seals, leading to leaks and component wear. Understanding the fluid's optimal temperature range and monitoring actual operating temperatures allows for the implementation of effective cooling systems (heat exchangers) and proactive fluid maintenance, preventing costly failures and ensuring system integrity.

Extending the components life span while operating within specified parameter limits is a fundamental aspect to consider. Contamination is the single largest cause of hydraulic system failure. Particles, even microscopic ones, act as abrasives, eroding precision surfaces in pumps, scoring cylinder bores, and causing valves to stick or leak. Knowing the required ISO cleanliness codes for a system and regularly monitoring fluid

contamination levels through particle counting allows for effective filtration strategies and proactive fluid replacement, dramatically extending the life of components. Selecting the correct fluid type is crucial. Regular fluid analysis, which includes testing for viscosity, acidity number, water content, and particle count, provides vital parameter data to determine when fluid needs filtration or replacement, preventing premature wear and system degradation.

Components are designed for specific operating envelopes. Consistently exceeding maximum pressure ratings or temperature limits will accelerate fatigue, degrade materials, and lead to early failure. Adhering to these parameters, which requires knowing them, is key to component longevity. Also, seals are critical for preventing internal and external leakage. Their lifespan is significantly affected by temperature, fluid compatibility, and pressure cycling. Understanding these parameters helps in selecting the right seal materials and ensuring the system operates within conditions that do not prematurely degrade them.

Engineers have to also make strategies for predictive maintenance, where parameter knowledge is essential. Modern fluid power systems often incorporate sensors that continuously monitor pressure, flow, temperature, and even contamination. Deviations from established baseline parameters (for example a gradual increase in pump case drain flow, a sudden drop in actuator speed, an unexplained rise in fluid temperature) can be early warning signs of impending component failure. This allows for predictive maintenance, where repairs are scheduled before a catastrophic breakdown occurs, and minimizing costly downtime. In summary, a deep understanding of hydraulic and pneumatic parameters is not just about optimizing performance, it's a fundamental requirement for building and operating systems that are inherently safe, reliable, and durable, protecting both assets and human lives.

Environmental impact

In an era defined by increasing environmental consciousness and sustainability targets, the principles governing the operation of hydraulic and pneumatic systems extend beyond mechanical efficiency. A deep understanding of fluid power parameters directly translates into environmental benefits, primarily through reduced energy consumption and waste materials. These two areas must be developed for industries seeking to lower their carbon footprint and adhere to responsible environmental practices.

The operational efficiency of hydraulic and pneumatic machinery is linked to energy consumption. In many industrial settings, the electricity powering these systems is generated, at least in part, from fossil fuels. Consequently, any reduction in energy demand directly contributes to a decrease in the combustion of coal, natural gas, or oil, thereby lowering the release of greenhouse gases into the atmosphere.

By meticulously optimizing system efficiency through a thorough understanding of parameters like pressure, flow, and power requirements, engineers can design and operate fluid power systems that consume only the necessary amount of energy. This involves correctly sizing pumps, motors, and actuators to match the application's actual load, preventing the wasteful practice of oversizing. Furthermore, implementing advanced control strategies, such as load-sensing pumps and variable frequency drives (VFDs) for electric motors, ensures that power is delivered precisely when and where it's needed, rather than constantly running at maximum capacity.

The aggregate effect of these optimizations is a tangible reduction in the industrial operation's overall electricity consumption. This directly translates into a lower carbon footprint, a quantifiable measure of the greenhouse gases emitted. For businesses, this not only aligns with

broader environmental goals of sustainability and climate change mitigation but also helps in meeting increasingly common regulatory requirements for energy efficiency. In many areas, energy audits and efficiency standards are becoming mandatory, and optimized fluid power systems play a critical role in achieving compliance and avoiding potential penalties. Beyond regulatory compliance, the public and investor scrutiny on corporate environmental responsibility makes energy efficiency a key differentiator for forward-thinking companies.

Beyond energy consumption, the integrity of fluid power systems brings a direct impact to resource utilization. Leaks, whether of compressed air in pneumatic systems or hydraulic fluid in hydraulic systems, represent a significant and often underestimated source of waste with both economic and environmental repercussions. In pneumatic systems, air leaks are a direct and immediate waste of energy. The compressed air that escapes through faulty seals, hoses, or fittings has already consumed a substantial amount of electricity during its compression. Every cubic foot of leaked air represents wasted energy that the compressor expended, leading to higher electricity bills and a larger carbon footprint. Identifying and rectifying these leaks through systematic leak detection programs, often guided by an understanding of pressure drops and flow rates, is a straightforward yet highly effective way to conserve energy and reduce operational costs.

In hydraulic systems, the consequences of leaks are even more pronounced. Hydraulic fluid, often petroleum-based or synthetic, is an expensive consumable. Leaks not only result in the direct loss of this valuable fluid but also pose significant environmental risks. Uncontained spills can contaminate soil and water sources, harming ecosystems and potentially requiring costly remediation efforts. Proper containment and disposal of leaked fluid are crucial to prevent such contamination. A

thorough knowledge of pressure and flow parameters is instrumental in preventing and addressing these issues. Regular monitoring for abnormal pressure drops or unexplained fluid loss can indicate internal or external leaks. By understanding how fluid should behave under specific operating conditions, technicians can pinpoint the source of a leak, whether it's a worn seal, a cracked hose, or a loose fitting, and implement timely repairs. This proactive approach conserves expensive hydraulic fluid, minimizes the risk of environmental contamination, and contributes to a safer, more sustainable industrial environment.

As a result, careful management of fluid power parameters offers a potent pathway to achieving significant environmental benefits. By prioritizing energy efficiency and rigorously preventing leaks, industries can substantially lower their carbon footprint, conserve valuable resources, and demonstrate a commitment to environmental stewardship, all while improving operational profitability.

Modern systems

Despite the undeniable importance of parameter knowledge, its acquisition and application are not without challenges. Modern fluid power systems are becoming increasingly complex, integrating with advanced control technologies and demanding a higher level of expertise. However, this complexity also paves the way for exciting future developments.

In the rapidly advancing landscape of industrial automation, contemporary hydraulic and pneumatic systems have transcended their simpler predecessors, evolving into highly sophisticated networks of interconnected components. No longer just about brute force, modern fluid power leverages cutting-edge technologies such as proportional valves, servo controls, load-sensing pumps, and intricate control algorithms to achieve unprecedented levels of precision, efficiency, and

adaptability. However, this leap in capability comes with a corresponding increase in complexity, demanding a far deeper and more holistic understanding of the interplay of multiple parameters than ever before. Proportional valves, for instance, are a cornerstone of modern fluid power, allowing for continuous and nuanced control of fluid flow and pressure, rather than simple on/off operation. Unlike traditional directional valves, proportional valves can vary the opening of their orifices, enabling precise control over cylinder speed and motor torque. Similarly, servo controls take this precision to an even higher level, often incorporating feedback loops to achieve exact positioning and velocity control. These systems utilize sophisticated electronics and sensors to constantly monitor output and adjust input, ensuring that the system operates precisely as commanded.

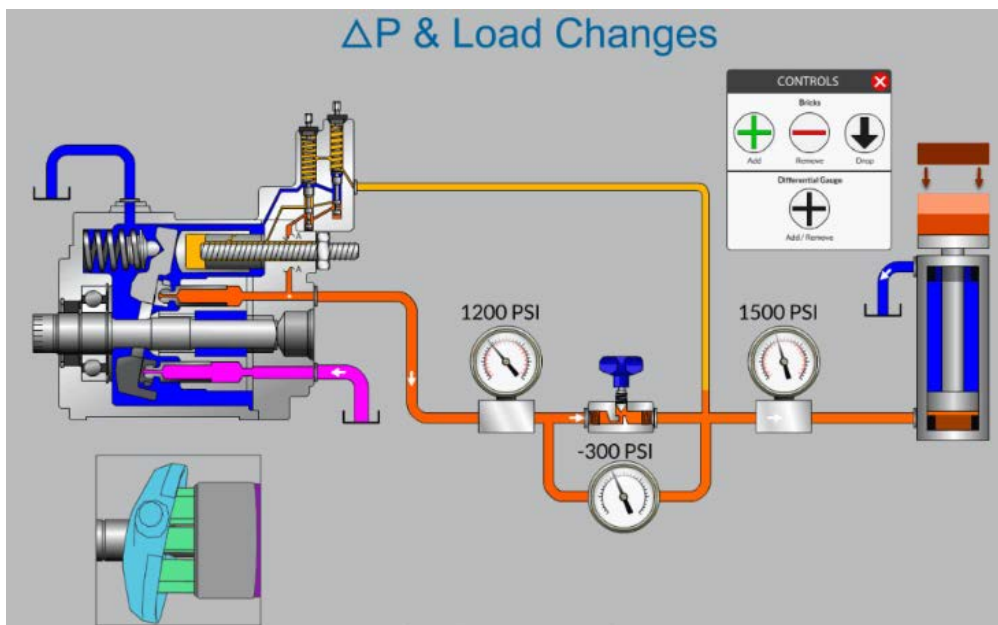


Fig.3. Load sensing system by CD Industrial Group Inc. [31]

The integration of load-sensing pumps (fig.3.) further exemplifies this drive for efficiency. Traditional fixed-displacement pumps deliver a constant flow regardless of the actual demand, often wasting energy by pumping more fluid than necessary. Load-sensing pumps, however,

adjust their displacement based on the actual load and flow requirements of the system. This not only conserves energy but also reduces heat generation and extends component life. When combined with proportional valves and servo controls, load-sensing pumps create a dynamic and highly efficient system that precisely matches power output to demand. However, the true challenge lies in the intricate control algorithms that orchestrate these components. These algorithms process data from numerous sensors—pressure transducers, flow meters, position encoders—and issue commands to the valves and pumps, often in real-time. From proportional integral derivative control loops to more advanced adaptive or predictive control strategies, these algorithms are designed to optimize performance, minimize energy consumption, and ensure stability across varying operating conditions.

The sheer sophistication of these systems means that the traditional approach of analyzing individual parameters in isolation is no longer sufficient. A change in one parameter, seemingly minor, can initiate a chain reaction of cascading effects throughout the entire system. For example, an adjustment to the setpoint of a proportional valve might alter the flow rate, which in turn could affect the pressure drop across a downstream component, influence the response of a load-sensing pump, and ultimately impact the accuracy of a servo-controlled actuator. Without a holistic understanding of how these parameters interrelate, troubleshooting becomes a daunting task, and optimization remains elusive.

Therefore, for engineers and technicians working with contemporary hydraulic and pneumatic systems, a deep comprehension of the interconnectedness of pressure, flow, viscosity, temperature, component characteristics, and control logic is paramount. It requires not just knowing the individual functions of each component but also understanding their

dynamic interaction under varying loads and operating conditions. This comprehensive perspective is essential for designing robust and efficient systems, diagnosing complex faults, and maximizing the performance and longevity of these vital industrial assets. In essence, the future of fluid power lies in mastering the intricate symphony of its parameters

Conclusions

In conclusion, the dynamic world of industrial operations, hydraulic and pneumatic machines stand as the backbone of countless processes, powering everything from manufacturing assembly lines to heavy construction equipment. The efficiency, safety, and longevity of these critical systems hinge entirely on one fundamental principle: a comprehensive understanding of their underlying parameters. This knowledge isn't just a technical detail; it's the very cornerstone of effective design, secure operation, and sustainable maintenance within the realm of fluid power. At its heart, fluid power relies on the precise manipulation of liquids and gases to transmit force and motion. This manipulation is governed by a set of interconnected parameters, including pressure, flow, and viscosity. Pressure, the force exerted per unit area, dictates the power and force a system can generate. Flow, the volume of fluid moving through a system over time, determines speed and responsiveness. Viscosity, a fluid's resistance to flow, influences friction losses and energy efficiency. Grasping these fundamental definitions, along with their intricate interplay within complex hydraulic and pneumatic circuits, directly impacts every aspect of a fluid power application.

This knowledge empowers engineers and technicians to create systems that are perfectly attuned to their intended tasks. Without it, there's a significant risk of oversizing or undersizing components. An oversized

system wastes energy, incurs higher initial costs, and often operates inefficiently, leading to accelerated wear and tear. Conversely, an undersized system struggles to meet demand, leading to poor performance, frequent breakdowns, and premature failure. By understanding parameters, professionals can precisely control speed, force, and position, ensuring optimal performance, consistent product quality, and maximum operational output.

Beyond efficiency, a profound understanding of hydraulic and pneumatic parameters forms the bedrock of safety protocols. Uncontrolled pressure, insufficient flow, or incorrect fluid selection can lead to catastrophic failures, endangering personnel and causing extensive damage to equipment. Knowledge of these parameters allows for the implementation of appropriate safety measures, including pressure relief valves, flow control mechanisms, and proper filtration, all of which mitigate risks and protect both human life and valuable assets. Furthermore, this expertise is crucial for proactive maintenance and informed troubleshooting. Recognizing deviations in parameters often provides early warning signs of impending issues, enabling timely interventions that extend the lifespan of expensive components and prevent costly downtime. The economic advantages of this specialized knowledge are substantial. By optimizing system design and operation, businesses can realize significant savings through reduced energy consumption. Efficiently sized and operated systems consume less power, directly impacting utility bills. Minimized downtime, a direct result of proactive maintenance and effective troubleshooting, translates into uninterrupted production and increased profitability. Moreover, optimized component longevity reduces replacement costs and extends the return on investment on machinery. From an environmental perspective, this expertise fosters sustainability by reducing resource waste, minimizing

leaks, and ultimately decreasing the energy footprint of industrial operations.

As industries evolve, becoming increasingly automated and data-driven, the importance of parameter knowledge will only amplify. The integration of the Internet of Things, Artificial Intelligence (AI), and predictive analytics will rely heavily on accurate parameter data for real-time monitoring, intelligent control, and proactive maintenance strategies. For anyone involved in the design, operation, or maintenance of hydraulic and pneumatic machinery, a profound understanding of these parameters transcends mere technical skill. It is a strategic imperative that ensures efficiency, reliability, safety, and profitability in an ever-evolving industrial world. It is, quite simply, the indispensable knowledge that underpins the success of fluid power.

1. DENSITY MEASUREMENT - DIRECT METHOD

1.1. Theoretical aspects

Determining the density of a liquid, by using the direct method, can be done by simultaneously weighting a volume of liquid, by using a graduated cylinder, and finding its mass using a precision scale as shown in the figure bellow (fig. 1.1).

Fig.1.1. Graduated tube and electronic scale used for determining the density

The density of a hydraulic liquid represents the ratio between the mass and volume of the liquid and is determined by the following equation:

$$\rho = \frac{m}{V} \left[\frac{kg}{m^3} \right] \quad [1.1]$$

where: ρ [kg/m^3] represents the density of the fluid;

m [kg] represents the mass of the fluid;

V [m^3] represents the volume of the fluid.

To increase the accuracy of the measurement, several determinations are made. The results will be processed on a statistical basis, according to the statistical processing procedure of experimental data.

The result can be expressed in the following form:

$$x = \bar{x} \pm t \cdot \bar{\sigma} = x_{min} \cdots x_{max} \quad [1.2]$$

1.2. Purpose of the experiment

The aim of this experiment is to determine the density of hydraulic fluids using the direct measurement method.

1.3. Experimental procedure

- Place an empty graduated glass cylinder on a scale;
- Find the mass of the empty cylinder by weighing it, or use the TARE function of the scale, which allows you to strictly determine the mass of the liquid by first weighing the empty graduated cylinder and setting the scale to 0 before adding the liquid;
- Add inside the cylinder a volume of liquid whose density must be determined;

- Measure the volume of the liquid using the graduated scale;
- Record the value for the volume in Table 1.2;
- Weigh the filled cylinder;
- Record the value for the mass of the liquid in Table 1.2;
- Set the scale to 0 (zero) using the TARE function;
- Add a new volume of liquid, above the existing one;
- Measure the new volume of liquid using the graduated scale;
- The difference between the current value for the volume and the previously recorded one is computed;
- Record the volume difference in table 1.2;
- Weigh the filled cylinder again;
- Record the value of the liquid mass in table 1.2;
- Set the scale to 0 using the TARE function;
- Add another volume of liquid to the existing one;
- Measure the volume of the liquid using the graduated scale;
- Compute the difference between the value for the current volume of fluid and the previous one in table 1.2;
- Weigh the filled cylinder;
- Record the value of the liquid mass in table 1.2;

To increase the accuracy of the measurement, several determinations are made and recorded in the provided data table.

The density of the hydraulic fluid is computed by using the following equation:

$$\rho = \frac{m}{V} \left[\frac{kg}{m^3} \right]$$

Computed values are written in table 1.2.

The results will be processed on a statistical basis, according to the Procedure for Statistical Processing of Experimental Data.

1.4. Statistical data processing

1.4.1. Average value or arithmetic mean

If a series of n measurements is made on a physical quantity X , under the same experimental conditions, we get the values X_1, X_2, \dots, X_n .

The arithmetic mean is determined with the formula:

$$\bar{X} = \frac{1}{n} \cdot \sum_{i=1}^n X_i \quad [1.3]$$

The average value is important in estimating the accuracy of the measurements. Often, this average value is adopted as a reference quantity. In the case of a very large series of measurements ($n \rightarrow \infty$), the average value \bar{X} tends to the true value of the measured quantity.

1.4.2. Square root mean

To calculate the value of the square root mean we use:

$$X_p = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^n X_i^2} \quad [1.4]$$

1.4.3. Standard deviation

The mean square deviation is a quantity used in the processing of experimental data and estimates the standard deviation. The calculation formula is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}} = \sqrt{\frac{\sum_{i=1}^n d_i^2}{n - 1}} \quad [1.5]$$

1.4.4. Errors processing

The root mean square error of the mean is computed with the following formula:

$$\bar{\sigma} = \frac{\sigma}{\sqrt{n}} \quad [1.6]$$

To express the result in the form of an interval of the determined quantity, the Student distribution parameters can be used (table 1.1).

In this sense, the confidence level is chosen. If there is no recommendation or indication, the confidence level $P = 0.95$ is recommended. For a number n of determinations performed, the corresponding $t(n, P)$ value is extracted from the table with the Student distribution parameters.

The result can be expressed in the following form:

$$x = \bar{x} \pm t \cdot \bar{\sigma} = x_{min} \cdots x_{max}$$

Table 1.1. Student distribution parameters.

n	P						
	0,683	0,900	0,95	0,955	0,99	0,997	0,999
2	1,83	6,31	12,71	13,97	63,66	387,0	636,6
3	1,32	2,92	4,30	4,53	9,92	19,21	31,60

4	1,20	2,35	3,18	3,31	5,84	9,22	12,94
5	1,14	2,13	2,78	2,87	4,60	6,62	8,61
6	1,11	2,02	2,57	2,65	4,03	5,51	6,86
7	1,09	1,91	2,45	2,52	3,71	4,90	5,90
8	1,08	1,90	2,36	2,43	3,50	4,53	5,41
9	1,07	1,86	2,31	2,38	3,3	4,28	5,04
10	1,06	1,83	2,26	2,33	3,25	4,09	4,78
11	1,05	1,81	2,23	2,30	3,17	3,96	4,59
12	1,05	1,79	2,20	2,27	3,11	3,86	4,44
13	1,04	1,78	2,18	2,24	3,05	3,77	4,32
14	1,04	1,77	2,16	2,22	3,01	3,71	4,22
15	1,04	1,76	2,14	2,20	2,98	3,64	4,14
16	1,03	1,75	2,13	2,18	2,95	3,59	4,07
17	1,03	1,74	2,12	2,17	2,92	3,54	4,01
18	1,03	1,74	2,11	2,16	2,90	3,51	3,96
19	1,03	1,73	2,10	2,15	2,88	3,48	3,92
20	1,03	1,73	2,09	2,14	2,86	3,45	3,88
∞	1,00	1,64	1,96	2,00	2,58	3,00	3,29

Table nr.1.2. Experimental data

Measured values	Computed values
-----------------	-----------------

n Number of tests	V Recorded volume on the graduated scale [cm ³]	m Mass on the scale [g]	ΔV Volume of fluid $\Delta V = V_n - V_{n-1}$ [cm ³]	ρ Density $\rho = m / \Delta V$ [g/cm ³]	$\bar{\rho}$ [g/cm ³]	σ_ρ [g/cm ³]	$\bar{\sigma}_\rho$ [g/cm ³]
1.							
2.							
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2. DENSITY MEASUREMENT – HARE'S TUBE METHOD

2.1. Theoretical aspects

The density of a liquid can also be determined by comparing its density with the density of a standard liquid (a known density). The standard or reference fluid is usually water or glycerine.

For this method, the Hare Tube is used. The Hare Tube's measuring principle consists in applying the same hydrostatic pressure to two columns of fluid of different densities.

The Hare Tube is a device made of a graduated tube, in the shape of an inverted U, which has a connection socket for a vacuum gun (mini vacuum pump) at the top, figure 2.1.

The two tubes are each placed in a beaker; one beaker contains the liquid whose density is to be determined, and the other one contains the standard liquid, whose density is known (e.g. distilled water).

Using a small vacuum pump, a drop of pressure is created in the upper part of the tubes, causing the two liquids to rise. The liquid columns will have different heights.

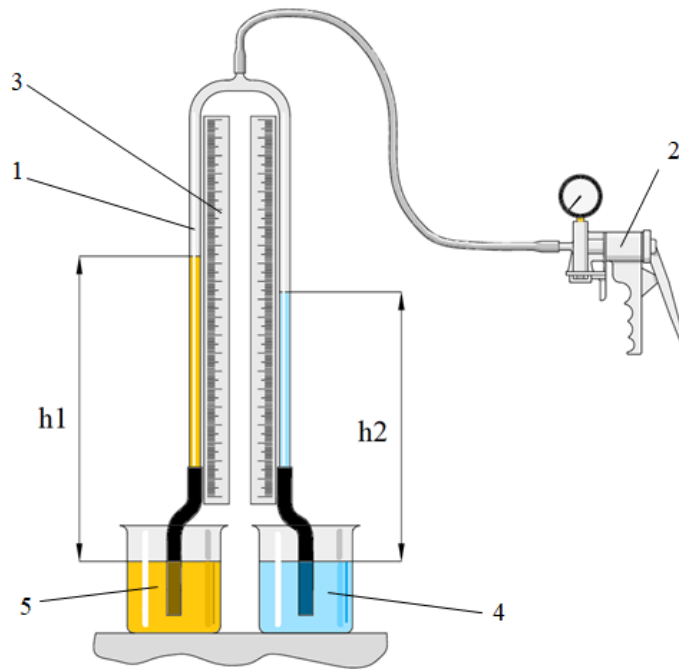


Fig.2.1. Hare's tube method for determining the density of a fluid.

where: 1 is Hare's tube; 2 represents the Vacuum pump; 3 is a graduated scale; 4 is the reference liquid (distilled water); 5 is the test liquid.

The density of the liquid is calculated using the formula:

$$p_v + \rho_w \cdot g \cdot h_w = p_v + \rho_l \cdot g \cdot h_l = p_{at} \quad [2.1]$$

From the equation above, the density of the liquid is:

$$\rho_l = \rho_w \cdot \frac{h_w}{h_l} \left[\frac{kg}{m^3} \right] \quad [2.2]$$

where ρ_l represents the unknown density of the unknown fluid;

ρ_w represents the reference density (eg. distilled water);

h_w represents the height of the reference column of fluid (eg. distilled water);

h_l represents the height of the unknown fluid.

To increase the accuracy of the measurement, several determinations are made and recorded. The results will be processed on a statistical basis, according to the Procedure for Statistical Processing of Experimental Data.

The result can be expressed in the following form:

$$x = \bar{x} \pm t \cdot \bar{\sigma} = x_{min} \cdots x_{max} \quad [2.3]$$

2.2. Purpose of the experiment

The purpose of this experiment is to determine the density of an unknown fluid by using Hare’s tube method.

2.3. Experimental procedure

- Two glasses are placed on an absolutely horizontal surface, under the soft ends of each tube (check with level ruler) otherwise adjust the horizontality of the table (stand);
- Fill one glass is filled with distilled water;
- The other glass is filled with the liquid whose density must be determined;
- With the vacuum pump, a vacuum is created in the upper part, the two liquids start rising unevenly on the two arms;
- The liquid is filled into the basic containers in such a way that the level in both vessels is the same and the ends of the hoses (soft ends) are immersed in the liquid;

- The heights of both columns are read on the tubes;
- The height of the water column and the height of the liquid column whose density must be determined are recorded in table 2.2;
- The valve of the vacuum pump is opened to lower the liquids into the glasses;
- The operation is repeated.

The unknown density is computed using the following formula:

$$\rho_l = \rho_w \cdot \frac{h_w}{h_l} \left[\frac{kg}{m^3} \right]$$

The value used for the density of water is 1000kg/m³.

The calculated values are recorded in table 2.2.

The results will be processed on a statistical basis, according to the Procedure for Statistical Processing of Experimental Data, the density value being presented in the form of the interval:

$$x = \bar{x} \pm t \cdot \bar{\sigma} = x_{min} \cdots x_{max}$$

2.4. Statistical data processing

2.4.1. Average value or arithmetic mean

If a series of n measurements is made on a physical quantity X, under the same experimental conditions, we get the values X_1, X_2, \dots, X_n .

The arithmetic mean is determined with the formula:

$$\bar{X} = \frac{1}{n} \cdot \sum_{i=1}^n X_i \quad [2.4]$$

The average value is important in estimating the accuracy of the measurements. Often, this average value is adopted as a reference quantity. In the case of a very large series of measurements ($n \rightarrow \infty$), the average value \bar{X} tends to the true value of the measured quantity.

2.4.2. Square root mean

To calculate the value of the square root mean we use:

$$X_p = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^n x_i^2} \quad [2.5]$$

2.4.3. Standard deviation

The mean square deviation is a quantity used in the processing of experimental data and estimates the standard deviation. The calculation formula is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}} = \sqrt{\frac{\sum_{i=1}^n d_i^2}{n - 1}} \quad [2.6]$$

2.4.4. Errors processing

The root mean square error of the mean is computed with the following formula:

$$\bar{\sigma} = \frac{\sigma}{\sqrt{n}} \quad [2.7]$$

To express the result in the form of an interval of the determined quantity, the Student distribution parameters can be used (table 1.1).

In this sense, the confidence level is chosen. If there is no

recommendation or indication, the confidence level $P = 0.95$ is recommended. For a number n of determinations performed, the corresponding $t(n, P)$ value is extracted from the table with the *Student distribution parameters*.

The result can be expressed in the following form:

$$x = \bar{x} \pm t \cdot \bar{\sigma} = x_{min} \cdots x_{max}$$

Table 2.1. Student distribution parameters.

n	P						
	0,683	0,900	0,95	0,955	0,99	0,997	0,999
2	1,83	6,31	12,71	13,97	63,66	387,0	636,6
3	1,32	2,92	4,30	4,53	9,92	19,21	31,60
4	1,20	2,35	3,18	3,31	5,84	9,22	12,94
5	1,14	2,13	2,78	2,87	4,60	6,62	8,61
6	1,11	2,02	2,57	2,65	4,03	5,51	6,86
7	1,09	1,91	2,45	2,52	3,71	4,90	5,90
8	1,08	1,90	2,36	2,43	3,50	4,53	5,41
9	1,07	1,86	2,31	2,38	3,3	4,28	5,04
10	1,06	1,83	2,26	2,33	3,25	4,09	4,78
11	1,05	1,81	2,23	2,30	3,17	3,96	4,59
12	1,05	1,79	2,20	2,27	3,11	3,86	4,44
13	1,04	1,78	2,18	2,24	3,05	3,77	4,32
14	1,04	1,77	2,16	2,22	3,01	3,71	4,22
15	1,04	1,76	2,14	2,20	2,98	3,64	4,14
16	1,03	1,75	2,13	2,18	2,95	3,59	4,07
17	1,03	1,74	2,12	2,17	2,92	3,54	4,01
18	1,03	1,74	2,11	2,16	2,90	3,51	3,96
19	1,03	1,73	2,10	2,15	2,88	3,48	3,92
20	1,03	1,73	2,09	2,14	2,86	3,45	3,88
∞	1,00	1,64	1,96	2,00	2,58	3,00	3,29

Table nr.2.2 Experimental data

Measured values			Computed values				
n Number of readings	h_w water column height [mm]	h_l fluid column height [mm]	ρ_w Reference density [kg/m ³]	ρ_l Fluid density [kg/m ³]	$\bar{\rho}$ [kg/m ³]	σ_ρ [kg/m ³]	$\bar{\sigma}_\rho$ [kg/m ³]
1.							
2.							
3.							
4.							
5.							
6.							
7.							
8.							
9.							
10.							

3. VISCOSITY MEASUREMENT – OSWALD VISCOMETER

3.1. Theoretical aspects

The measurement of the viscosity of a liquid, by comparison with the viscosity of a standard fluid (usually distilled water), whose viscosity is known, can be done using the Oswald Viscometer.

Viscosity forces occur when the liquid flows, under the action of gravity, through a capillary. Therefore, if a given volume of liquid is passed through a special tube and the flow time is measured, then it can be compared with the time in which a liquid passed through its

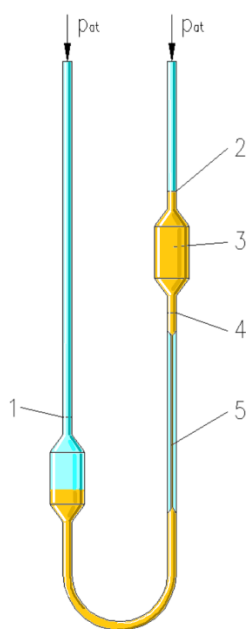


Fig.3.1. Oswald Viscometer

where: 1. filling mark; 2. measurement start mark; 3. volume; 4. measurement end mark; 5. capillary.

The Oswald Viscometer is similar to an indirect U-shaped piezometric tube, with unequal sections, open at both ends (figure 3.1). On one of the arms there is a calibrated capillary channel.

The device is suspended (to ensure its verticality) in a cylinder in which it can be immersed in a thermostated bath, in the case of tests intended to establish the dependence of viscosity on temperature.

Table 3.1. Viscosity ISO 3448, at 40°C

Nr.	Viscosity cat. ISO	Average Kinematic Viscosity [mm ² /s]	Kinematic viscosity- limit values [mm ² /s]	
			Min.	Max.
1	ISO VG 2	2,2	1,98	2,42
2	ISO VG 3	3,2	2,88	3,52
3	ISO VG 5	4,6	4,14	5,06
4	ISO VG 7	6,8	6,12	7,48
5	ISO VG 10	10	9,00	11,00
6	ISO VG 15	15	13,5	16,5
7	ISO VG 22	22	19,8	24,2
8	ISO VG 32	32	28,8	35,2
9	ISO VG 46	46	41,4	50,6
10	ISO VG 68	68	61,2	74,8
11	ISO VG 100	100	90,0	110
12	ISO VG 150	150	135	165
13	ISO VG 220	220	198	242
14	ISO VG 320	320	288	352
15	ISO VG 460	460	414	506
16	ISO VG 680	680	612	748
17	ISO VG 1000	1000	900	1100
18	ISO VG 1500	1500	1350	1650
19	ISO VG 2200	2200	1980	2420
20	ISO VG 3200	3200	2880	3520

The viscosity is proportional to the flow time, the constant of the device, k , depending on the geometric elements of the installation, therefore, having a unique value for a given installation.

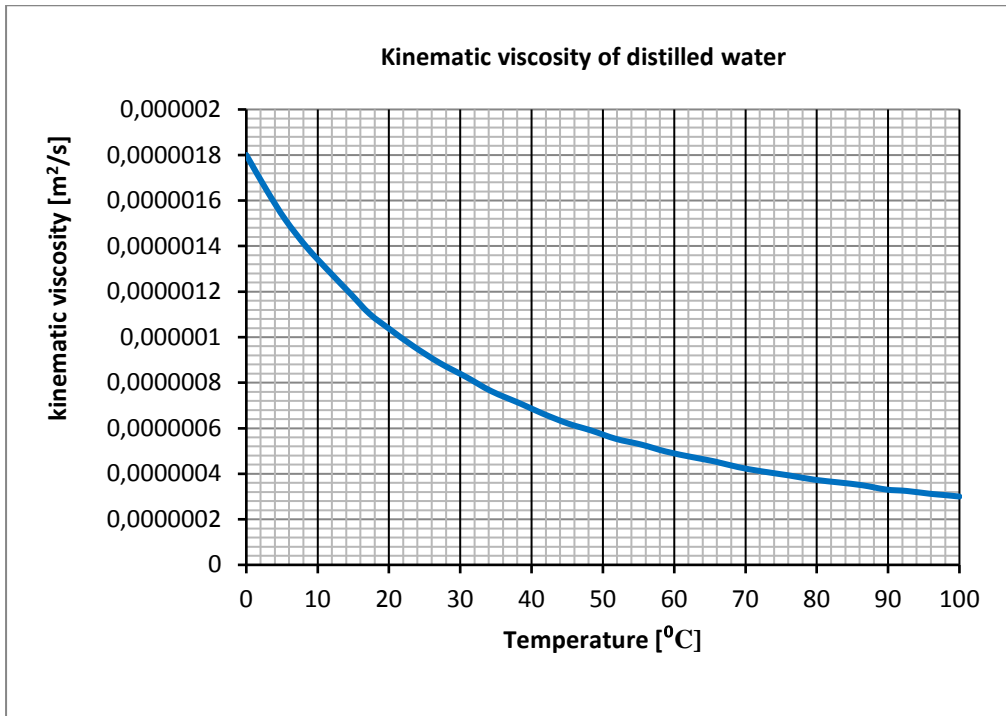


Fig. 3.2. Dependence of the kinematic viscosity of distilled water on temperature

The constant of the device is determined by the formula:

$$k = \frac{\nu_w}{t_{med w}} \quad [3.1]$$

where k represents the device constant;

ν_w represents the viscosity of distilled water, according to the graph in figure 3.2;

$t_{med w}$ represents the average flow time of distilled water for a given volume.

The viscosity of the liquid is determined by the formula:

$$\nu_l = k \cdot t_l \quad [3.2]$$

where ν_l represents the viscosity of the fluid;

k represents the device constant;

t_l represents the flow time of the liquid, for the same given volume.

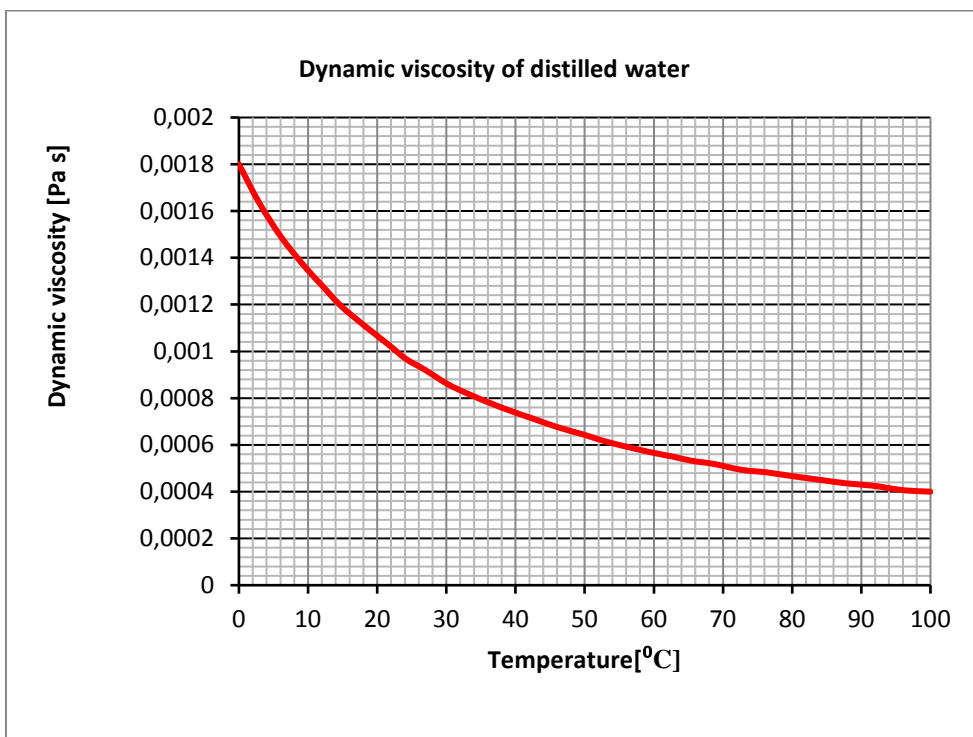


Fig. 3.3. Dependence of the dynamic viscosity of distilled water on temperature

With the value determined from the graph for the kinematic viscosity of distilled water at the test temperature, the result is:

$$\nu_w = k \cdot t_w \Rightarrow k = \frac{\nu_w}{t_w} \quad [3.3]$$

3.2. Experimental procedure

- Pour distilled water into a glass;
- Measure the temperature of the distilled water with a thermometer;
- Fill the viscometer with distilled water;
- Aspirate the distilled water through the capillary to above the reservoir at the top of the capillary, using the vacuum pump;
- Disconnect the vacuum pump;
- Find the time in which the distilled water flows through the capillary, descending between the two marks, drawn on the tube above and below that reservoir. The stopwatch is started when the surface of the column reaches the upper mark of the reservoir and is stopped when the surface of the water column reaches the lower mark of the reservoir;
- Record the time t_1 , in seconds, in Table 3.2;
- Refill the viscometer with distilled water;
- Find the second value for the time in which the distilled water flows through the capillary;
- Record the time t_2 , in seconds, table 3.2;
- Recharge the viscometer with distilled water;
- Find the third time during which the distilled water flows through the capillary;

- Record the time t_3 , in seconds, table 3.2;
- Drain the distilled water, clean and blow the device so that any water residues blocked in the capillary do not contaminate the next sample;
- Pour the liquid whose kinematic viscosity is to be determined into a glass;
- Load the viscometer with the liquid to be tested;
- Aspirate the liquid to be measured through the capillary to above the reservoir above the capillary, using the vacuum pump;
- Disconnect the vacuum pump;
- Find the time during which the liquid flows through the capillary, descending between the two marks, drawn on the tube above and below that reservoir. The timer starts when the surface of the column reaches the upper mark of the tank and stops when the surface of the water column reaches the lower mark of the tank;
- Determine from the curve of variation of kinematic viscosity with temperature based on this time, (figure 3.2) the value of kinematic viscosity ν of distilled water;
- Record the value of kinematic viscosity ν of water in table 3.2;
- Calculate the viscosity of the liquid using the presented formulas;
- Record the value of kinematic viscosity ν of the liquid, in table 3.2;

Table 3.2. Experimental data

	Temperature [°C]	Measured time(water) [s]			Average time t_m [s]	ν_w [m ² /s]	Device constant k [m ² /s ²]
		t_1	t_2	t_3			
Distilled water							
	Temperature [°C]	Measured flow time(fluid) [s]			ν_l [m ² /s]		
1.							
2.							
3.							
4.							
5.							
6.							

4. DYNAMIC VISCOSITY MEASUREMENT –

BROOKFIELD'S VISCOMETER

4.1. Theoretical aspects

For viscous hydraulic fluids where the flow time in the Ostwald viscometer would be very long, another type of viscometer can be used. This is the Brookfield rotational viscometer that measures the dynamic or absolute viscosity, η .

The Brookfield rotational viscometer principle is based on measuring the torque required to rotate a rotating body immersed in a liquid. The torque is proportional to the viscosity of the liquid.

The Brookfield viscometer type RVB 1 is a rotational viscometer that measures the torque required to rotate a body immersed in a fluid. The body is driven by an electric motor through a calibrated spring; the deformation of the spring is measured with an indicator and a dial.

The operating principle of the Brookfield viscometer is highlighted in figure 4.1. The motor 1 drives, through the spring 4, the rotating body 6, which is immersed in the fluid 5, the viscosity of which is measured. The degree of deformation of the spring 4, which transmits the rotational moment from the motor 1 to the rotating body 6, is highlighted by the indicator needle 3 and the scale on the dial 2.

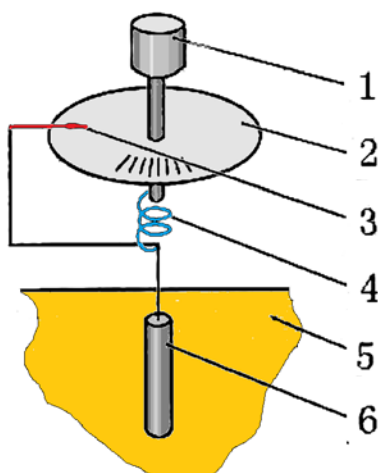


Fig.4.1. Working principle of the Brookfield rotational viscometer

By using a multi-speed transmission and interchangeable bodies, a variety of viscosity ranges can be measured, thus increasing the versatility of the instrument. For a given viscosity, the resistance to flow, indicated by the degree to which the spring deforms, is proportional to the speed of rotation of the body and depends on its size and shape.

The Brookfield rotational viscometer (figure 4.2) has a motor 1 that drives the rotating body 3, which is immersed in the test liquid in the beaker 7, to the mark 4, by means of a spring. The torque required to rotate the rotating body is displayed on the dial 2. The motor moves vertically on the column of the stand 6, using the positioning mechanism 5.

It is intended for measuring the viscosity of various liquids such as oils, chemicals, paints, foods, having a dynamic viscosity between 10-105 mPa s (cP). The electric motor can drive the rotating body at four speeds: 6, 12, 30 and 60 revolutions per minute.



Fig.4.2. Brookfield rotational viscometer

The rotating bodies are made of stainless steel and come in several sizes (figure 4.3). The viscosity calculation is done using the k coefficient specific to each rotating body. The values of the k coefficient depending on the speed and number of rotating bodies are presented in table 4.1.



Fig.4.3. Rotational bodies of the Brookfield rotational viscometer

Table 4.1. Values of k , function of speed and rotating body

Speed level	Rotational bodies number			
	1	2	3	4
6	10	50	200	1000
12	5	25	100	500
30	2	10	40	200
60	1	5	20	100

The dynamic viscosity measured with the Brookfield apparatus is determined by using the following formula:

$$\eta = k \cdot \alpha \quad [4.1]$$

where: $\eta [mPa]$ - dynamic viscosity;

k - coefficient depending on the speed and number of rotating body;

α – recorded value on the device dial.

To increase the accuracy of the measurement, several determinations are made and recorded in the table. The results will be processed on a statistical basis, according to the Procedure for Statistical Processing of Experimental Data.

The result can be expressed in the following form:

$$x = \bar{x} \pm t \cdot \bar{\sigma} = x_{min} \cdots x_{max} \quad [4.2]$$

4.2. Purpose of the experiment

The aim of this laboratory is to determine the dynamic viscosity of hydraulic fluids using high precision methods.

4.3. Experimental procedure

- Pour the Hydraulic fluid whose viscosity is to be determined into a glass;
- Mount the selected rotating body to the device;
- Position the glass under the device;
- Lower the device so that the rotating body is immersed in the fluid in the glass. Lower the rotating body until the mark on the axis of the rotating body reaches the level of the free surface of the fluid;
- Choose a drive speed;
- Turn on the device;
- Read the indication on the device dial; If the device indication cannot be read because it is too small or too large, exceeding the scale, turn off the device and change the rotating body with a larger or smaller one, respectively.
- Record the value read in table 4, corresponding to the speed and number of the rotating body used;
- Compute the viscosity of the liquid using the presented formula;
- Make several determinations, with different speeds and rotating bodies;
- The obtained data is statistically processed;
- The final result is presented in the form of a tolerated value (or range of values).

4.4. Statistical data processing

4.4.1. Average value or arithmetic mean

If a series of n measurements is made on a physical quantity X , under the same experimental conditions, we get the values X_1, X_2, \dots, X_n .

The arithmetic mean is determined with the formula:

$$\bar{X} = \frac{1}{n} \cdot \sum_{i=1}^n X_i \quad [4.3]$$

The average value is important in estimating the accuracy of the measurements. Often, this average value is adopted as a reference quantity. In the case of a very large series of measurements ($n \rightarrow \infty$), the average value \bar{X} tends to the true value of the measured quantity.

4.4.2. Square root mean

To calculate the value of the square root mean, we use:

$$X_p = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^n x_i^2} \quad [4.4]$$

4.4.3. Standard deviation

The mean square deviation is a quantity used in the processing of experimental data and estimates the standard deviation. The calculation formula is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}} = \sqrt{\frac{\sum_{i=1}^n d_i^2}{n - 1}} \quad [4.5]$$

4.4.4. Errors processing

The root mean square error of the mean is computed with the following formula:

$$\bar{\sigma} = \frac{\sigma}{\sqrt{n}} \quad [4.6]$$

To express the result in the form of an interval of the determined quantity, the Student distribution parameters can be used (table 1.1).

In this sense, the confidence level is chosen. If there is no recommendation or indication, the confidence level $P = 0.95$ is recommended. For a number n of determinations performed, the corresponding $t(n, P)$ value is extracted from the table with the *Student distribution parameters*.

The result can be expressed in the following form:

$$x = \bar{x} \pm t \cdot \bar{\sigma} = x_{min} \cdots x_{max}$$

Table 4.2. Student distribution parameters

n	P						
	0,683	0,900	0,95	0,955	0,99	0,997	0,999
2	1,83	6,31	12,71	13,97	63,66	387,0	636,6
3	1,32	2,92	4,30	4,53	9,92	19,21	31,60
4	1,20	2,35	3,18	3,31	5,84	9,22	12,94
5	1,14	2,13	2,78	2,87	4,60	6,62	8,61
6	1,11	2,02	2,57	2,65	4,03	5,51	6,86
7	1,09	1,91	2,45	2,52	3,71	4,90	5,90
8	1,08	1,90	2,36	2,43	3,50	4,53	5,41
9	1,07	1,86	2,31	2,38	3,3	4,28	5,04
10	1,06	1,83	2,26	2,33	3,25	4,09	4,78
11	1,05	1,81	2,23	2,30	3,17	3,96	4,59
12	1,05	1,79	2,20	2,27	3,11	3,86	4,44
13	1,04	1,78	2,18	2,24	3,05	3,77	4,32
14	1,04	1,77	2,16	2,22	3,01	3,71	4,22
15	1,04	1,76	2,14	2,20	2,98	3,64	4,14
16	1,03	1,75	2,13	2,18	2,95	3,59	4,07
17	1,03	1,74	2,12	2,17	2,92	3,54	4,01
18	1,03	1,74	2,11	2,16	2,90	3,51	3,96
19	1,03	1,73	2,10	2,15	2,88	3,48	3,92
20	1,03	1,73	2,09	2,14	2,86	3,45	3,88
∞	1,00	1,64	1,96	2,00	2,58	3,00	3,29

5. SURFACE TENSION MEASUREMENT – TRAUBE'S TUBE

5.1. Theoretical aspects

A method for determining surface tension is based on the observation that a liquid in a capillary tube flows out of it forming drops. The size of the drops formed at the end of the capillary depends on the surface tension of the liquid, namely, at the moment of rupture, the weight of the liquid drop is equal to the elastic tensile force of the "membrane" supporting the drop, denoted F . Let R be the inner or outer radius of the capillary, as the drop clings to its inner or outer walls, and σ the surface tension of the liquid.

The force exerted F along the circumference $2 \cdot \pi \cdot R$ is equal to the weight of the drop G at the moment of detachment:

$$F = 2 \cdot \pi \cdot R \cdot \sigma \text{ [N]} \quad [5.1]$$

$$G = \frac{m \cdot g}{n} \text{ [N]} \quad [5.2]$$

The surface tension is computed with the following formula:

$$\sigma = \frac{m \cdot g}{2 \cdot \pi \cdot R \cdot n} \left[\frac{\text{N}}{\text{m}} \right] \quad [5.3]$$

Using a relative method allows us to eliminate volume and radius measurements. To do this, we determine the number of droplets for a given volume of a fluid whose surface tension we know, then determine the

number of droplets for the same volume of liquid of the fluid whose surface tension we wish to determine.

Using the following equation we get the surface tension in the presented case:

$$\sigma = \frac{\rho \cdot n_0}{\rho_0 \cdot n} \cdot \sigma_0 \quad [5.4]$$

- ρ represents the density of the fluid;
- n is the number of droplets;
- σ represents the surface tension.

The parameters of the fluid whose surface tension we know are denoted with a subscript of 0, and the parameters of the fluid whose surface tension we wish to determine are denoted without a subscript.

5.2. Traube's Tube

The stalagmometer, also called the Traube tube (figure 5.1), consists of a small glass flask 1 which is fitted at the bottom with a capillary tube. The drain hole ends with a well-polished flat surface. The apparatus is fixed in a vertical position on a stand 4. The liquid drips into the beaker 2 which is on the scale 3.

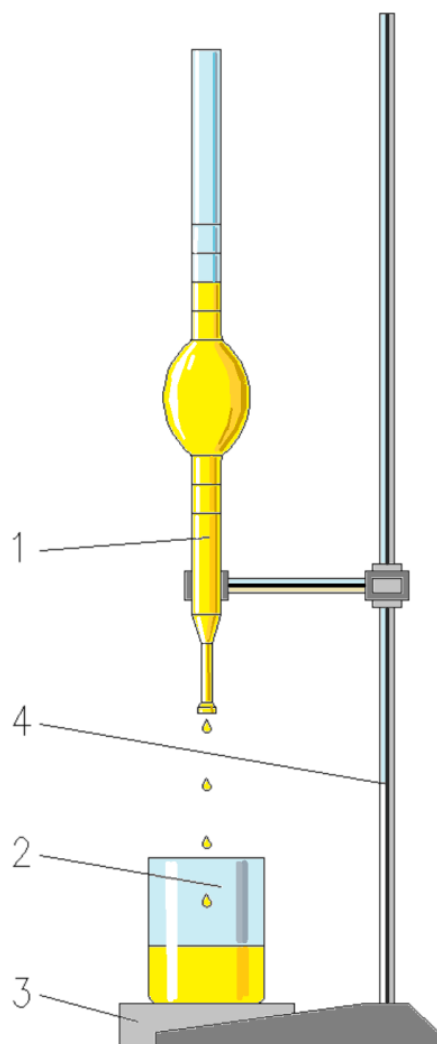


Fig.5.1. Stalagmometer

The variation of the surface tension σ of distilled water is shown in figure 5.2

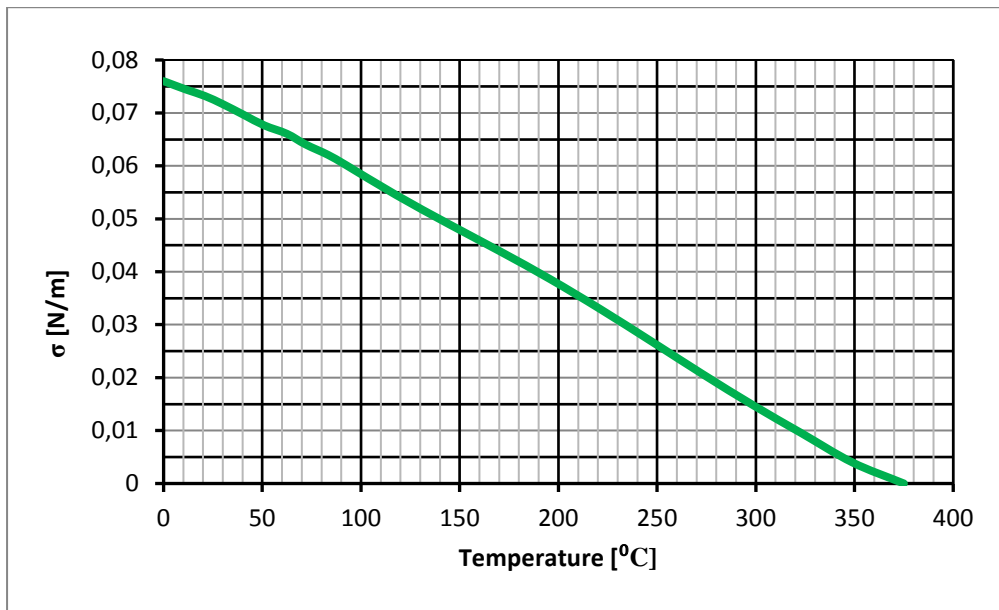


Fig. 5.2. Variation of surface tension of water with temperature

The density of distilled water is: 1000kg/m^3 .

5.3. Experimental procedure

- Measure the temperature;
- Fill the stalagmometer with distilled water;
- Place a dry glass under the capillary and let it fall, drop by drop, into the glass, counting a number of drops corresponding to the volume of the glass flask, n_0 ;
- Record the value n_0 for the distilled water in table 4.1;
- Clean the stalagmometer thoroughly;
- Fill the stalagmometer with the liquid to be measured;
- Place a dry glass under the capillary and let it fall, drop by drop, into the glass, counting a number of drops corresponding to the volume of the glass flask, n ;

- Determine the surface tension of the liquid in N/m using the formula;
- Repeat the determination three times.

Table 4.1. Experimental data

Nr.	Fluid	Temperature t [°C]	Number of droplets n	Density ρ [kg/m ³]	Surface tension σ [N/m]
1					
2					
3					
4					
5					

6. CONSTRUCTION OF HYDRAULIC AND PNEUMATIC DRIVE SCHEMES

6.1. Theoretical aspects

To create a hydraulic drive installation, several stages must be followed

Step 1:

In the first stage, the design theme, or the realization theme, of the hydraulic drive installation is analyzed. In this stage, the following must be established:

- start and stop conditions;
- stop conditions in case of failure;
- work phases and their sequence;
- interconditioning;
- adjustments of working speeds or forces;
- signaling of operation or failure;
- timings, etc.

Step 2:

Based on this information, an inventory of the hydraulic equipment required to create the drive scheme is made. The list must include the hydraulic elements needed:

- liquid tank;
- filter;
- hydraulic pump;

- hydraulic motor (cylinder);
- safety valve, pressure limiting valve, direction valve;
- control distributor;
- throttle;
- pressure gauge;
- stroke limiter, sensors, etc.

Step 3:

All the necessary elements to create the diagram are represented by symbols. The elements can be positioned on the diagram in two ways:

- Topographic layout;
- Level layout;

In the topographic layout, the elements are positioned in the diagram in such a way as to suggest the actual layout in the installation. The topographic layout is used in the case of simple diagrams, with a small number of elements, where the circuits can be easily followed.

In the level layout, the elements are positioned without taking into account the actual location in the installation, so that the energy and information flow goes from the lower part of the diagram to the upper part, and the sequences of the operating cycle or the work phases run from left to right

Step 4:

After placing the hydraulic elements necessary to create the hydraulic

drive scheme, the hydraulic connections between these elements are made so that the requirements of the design theme, or the installation theme, are met.

6.2. Example of a hydraulic diagram

The following example shows how to create a hydraulic diagram of a drive system.

Hydraulic diagram

The purpose is to create a diagram of a hydraulic lifting system that in Phase 1 raises the load to the maximum height, and in Phase 2 ensures the lowering under its own weight.

Step I. We need to identify the required elements for the system:

1. a single-acting cylinder;
2. a three-way, two-phase distributor;
3. a safety valve;
4. a hydraulic pump;
5. a hydraulic tank.

The necessary elements are placed as showed in the figure bellow.

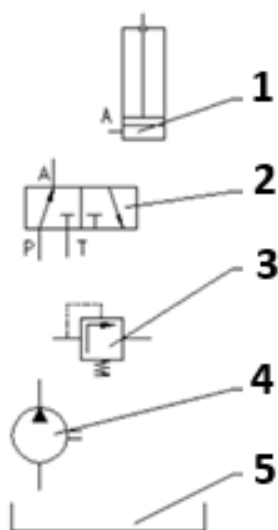


Fig.6.1.

Setp II. The connections are made (figure 6.2).

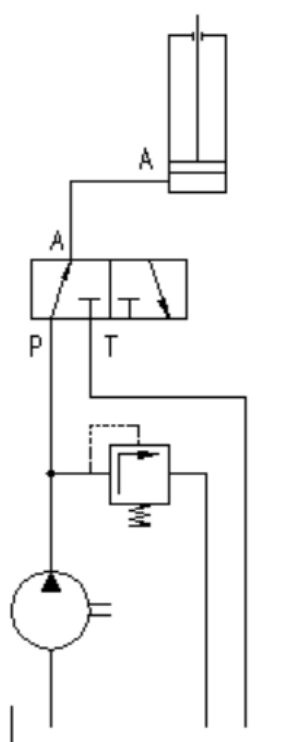


Fig.6.2. Hydraulic dyagram

Step III. Description of the working phases of the hydraulic scheme created (figure 6.3).

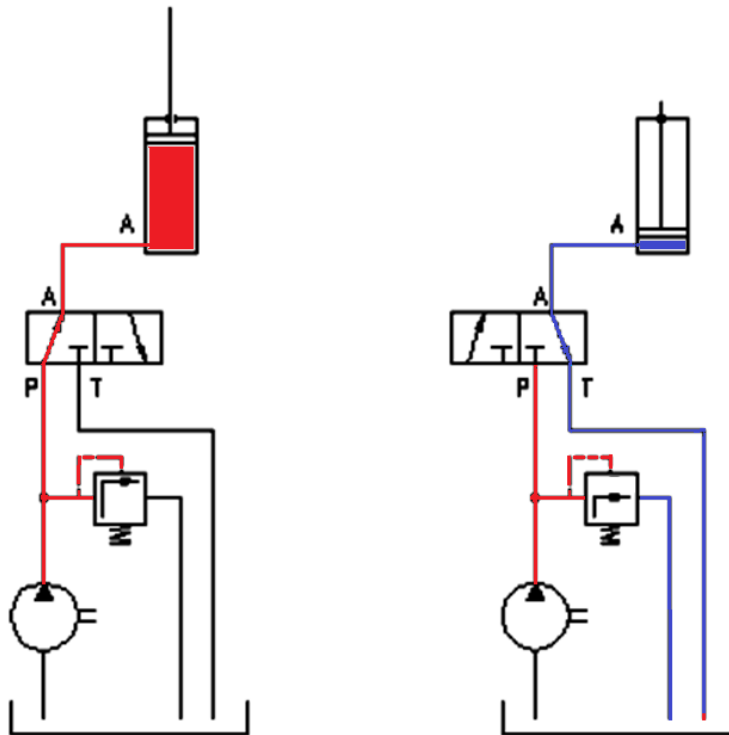


Fig.6.3. The two phases of the actuation diagram

Pneumatic diagram

To create a scheme of a pneumatic lifting installation that in Phase 1 will lift the load to the maximum height, and in Phase 2 will ensure the descent under its own weight.

Step I. Identify the necessary elements:

1. Compressor;
2. Filter;
3. Tank;
4. Pressure valve;

5. Pneumatic attenuator;
6. Distributor;
7. Pneumatic cylinder.

Step II. Place the necessary elements (figure 6.4) to create a actuation
(figure 6.5):

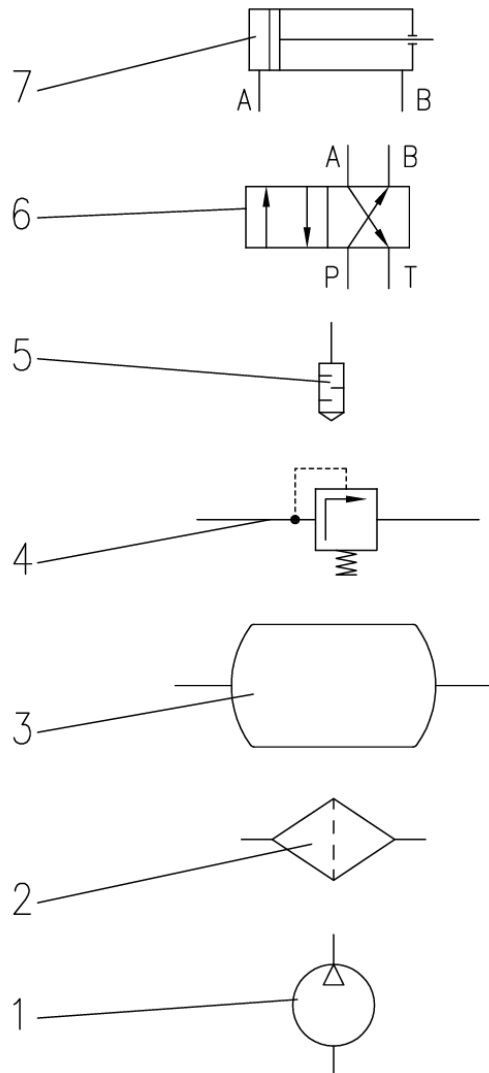


Fig.6.4. Pneumatic diagram elements



Step III. Description of the working phases of the completed pneumatic scheme (figure 6.6):

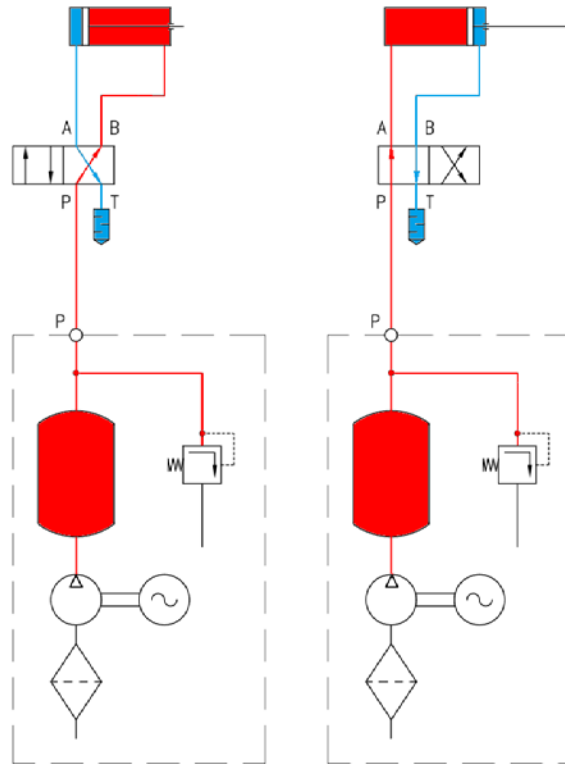


Fig.6.6. The two phases of pneumatic actuation

6.3. Purpose of the experiment

The paper aims to familiarize students with the preparation of functional schemes of hydraulic and pneumatic drives. The goal is for them to acquire the methodology, principles and rules of representation that underlie the creation of hydraulic and pneumatic drive schemes. Hydraulic and pneumatic schemes must be created in such a way as to ensure the easiest possible description and understanding of their operation.

6.4. Experimental procedure

- Draw a diagram of a hydraulic/pneumatic system that, using a double-acting hydraulic cylinder, lifts a load vertically. The diagram must have three working phases. In the first phase, the cylinder must lift the load. In the second phase, the load must be stationary. In this position, the load must not be moved manually. In the third phase, the cylinder must lower the load. The diagram must contain a pressure gauge on the supply line.
- This previously drawn diagram must ensure that in the event of damage to the pipe that ensures the lifting of the cylinder, the load does not accidentally lower.
- Draw a diagram of a hydraulic/pneumatic system that, using a hydraulic cylinder, moves a load horizontally. The diagram must have three working phases. In the first phase, the cylinder must push the load to the right. In the second phase, the load must be stationary. In this position, the load must not be moved manually. In the third phase, the cylinder must move the load to the left.
- This previously made scheme must be able to be manually moved horizontally in the second phase when the load is stationary.
- The same scheme must have four phases. In the first phase, the cylinder must push the load to the right at maximum speed. In the second phase, the cylinder must push the load to the right at

reduced speed. In the third phase, when the load is stationary, it must be manually moved horizontally. In the fourth phase, the cylinder must move the load to the left at maximum speed. The scheme must contain a pressure gauge on the supply line.

- Make a scheme of a hydraulic/pneumatic installation with two distributors that actuate two double-acting hydraulic cylinders. The first cylinder is controlled by distributor 1, which has two phases. In the first phase of distributor 1, the first cylinder lifts a load vertically, and in the second phase of distributor 1, the first cylinder lowers the load. Distributor 2 has three phases and controls the second cylinder. The distributor 2 must operate the second cylinder only when the load is held up by the first cylinder. Thus, in the first phase of the distributor 2 the second cylinder must move the load to the right, in the second phase the load must be locked horizontally, and in the third phase the hydraulic cylinder must move the load to the left. The diagram must contain a pressure gauge indicating the working pressure of the liquid for the first distributor and a pressure gauge indicating the working pressure of the liquid for the second distributor.
- Create your own actuation diagram, which differs from the ones presented and create the physical model using the provided elements in the laboratory.

7. WORKING PARAMETERS OF A HYDRAULIC OR PNEUMATIC CYLINDER

7.1. Theoretical aspects

In general, hydropneumatic machines consist of a cylinder and a piston that slides inside this cylinder (fig 7.1).

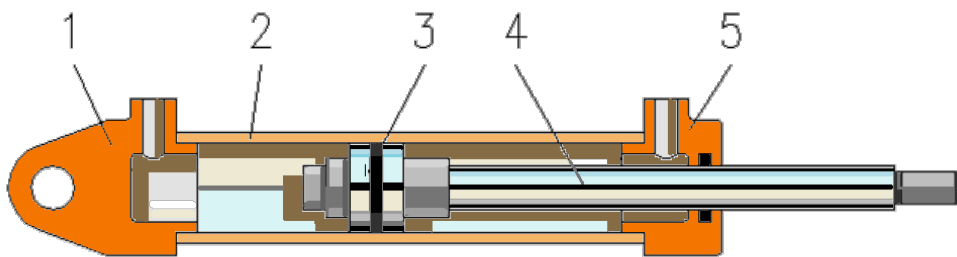


Fig.7.1. Component elements of a hydropneumatic drive

Where 1. cylinder base; 2. cylinder liner; 3. piston; 4. piston rod; 5. cylinder head.

The classification of hydropneumatic machines is made according to the number of directions in which the active element (piston) moves under the action of the pressure force and according to the construction of the element.

According to the way in which the fluid acts on the faces of the piston, achieving its displacement, hydropneumatic linear volumetric machines are classified into two classes:

- single-acting;
- double-acting.

- cylinders with a piston and a unilateral rod;
- cylinders with a plunger;
- telescopic cylinders.

Hydraulic/pneumatic sizing calculation, which determines the section, respectively the diameter of the piston and the rod so as to ensure a certain speed of movement; here it must be taken into account whether the motor is with a bilateral or unilateral rod:

$$S_1 = \frac{\pi \cdot D^2}{4} \quad [6.1]$$

$$S_2 = \frac{\pi \cdot (D^2 - d^2)}{4} \quad [6.2]$$

where: S_1 represents the area of the face of the piston without the rod;

S_2 represents the area on the face of the piston with the rod;

D is the diameter of the piston;

d is the diameter of the rod.

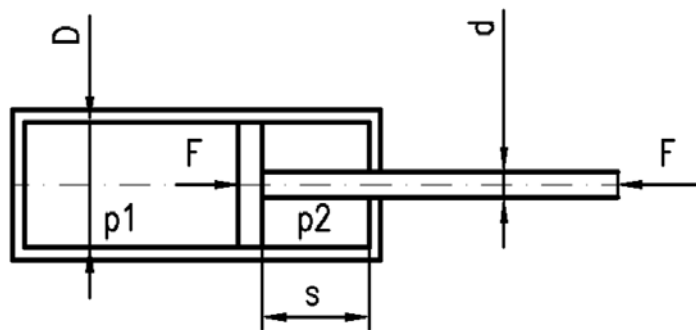


Fig.7.4. Elements of a hydropneumatic drive

The theoretical forces developed by the cylinders, for the two directions of movement, are:

$$F_1 = S_1 \cdot p = \frac{\pi \cdot D^2}{4} \cdot p \quad [6.3]$$

$$F_2 = S_2 \cdot p = \frac{\pi \cdot (D^2 - d^2)}{4} \cdot p \quad [6.4]$$

where: - F_1 is the force developed on the face of the piston without a rod;

- F_2 is the force developed on the face of the piston with a rod;

- p is the working pressure of the hydraulic fluid.

In a single-acting cylinder, the force developed by the piston is:

$$F_1 = S_1 \cdot p = \frac{\pi \cdot D^2}{4} \cdot p \quad [6.5]$$

In the single-acting cylinder, with a spring to return the piston, the developed forces are:

$$F_1 = S_1 \cdot p = \frac{\pi \cdot D^2}{4} \cdot p - k \cdot x \quad [6.6]$$

where: - k represents the elastic constant of the spring;

- x is the stroke of the piston.

The hydraulic/pneumatic computation to verify the section, which determines the supply pressure so that the useful force F_u overcomes the set of resisting forces, has the following steps:

$$F_u = \sum R \quad [6.7]$$

or

$$p_i \cdot S_1 - p_e \cdot S_2 - \sum R = 0 \quad [6.8]$$

where: - F_u is the useful force;

- p_i is the cylinder supply pressure;

- p_e is the cylinder return pressure;
- S_1 is the piston surface area on the supply side;
- S_2 is the piston surface area on the return side;
- ΣR is the sum of resisting forces.

The set of resisting forces represents the sum of the forces opposing the piston movement is:

$$\Sigma R = F_S + \Sigma F_{fr.s.} + \Sigma F_{fr.g.} \pm F_i \quad [6.9]$$

where: - F_S is the load to be transported;

- $\Sigma F_{fr.et}$ is the set of friction forces from the sealed areas;
- $\Sigma F_{fr.g}$ is the set of friction forces from the guides;
- F_i is the inertia force.

The pressure on the rodless face of the piston is computed with the equation:

$$p_1 = \frac{4 \cdot (F_S + \Sigma F_{fr.s.} + \Sigma F_{fr.g.} \pm F_i + p_2 \cdot S_2)}{\pi \cdot D^2} \quad [6.10]$$

and for the piston rod face, it is computed with the formula:

$$p_2 = \frac{4 \cdot (F_S + \Sigma F_{fr.s.} + \Sigma F_{fr.g.} \pm F_i + p_1 \cdot S_1)}{\pi \cdot (D^2 - d^2)} \quad [6.11]$$

Where: - p_1 is the pressure on face S_1 ;

- p_2 is the pressure on face S_2 .

In order to find the cylinder resistance we start from the hypothesis of a closed container of internal diameter D and wall thickness δ_c .

It is considered that between the diameter D and the wall thickness δ_c of

the cylinder we have the following condition:

$$\frac{D}{\delta_c} > 16 \quad [6.12]$$

so that:

$$\delta_c = \frac{p_a \cdot D}{2 \cdot \sigma_a} \quad [6.13]$$

where: - δ_c is the thickness of the wall;

- D is the inner diameter of the cylinder

- p_a is the maximum pressure in the cylinder;

- $\sigma_a = (500 \dots 800) \text{ [daN/cm}^2\text{]}$ is the maximum allowed value for the stress of the material from which the cylinder is made.

Buckling check of the cylinder rod.

One of the main stresses of the cylinder rod is the buckling load. The critical load at which buckling occurs is computed using the following equation:

$$F_{e\ cr} = \frac{\pi^2 \cdot E \cdot I}{l_f} \quad [6.14]$$

where: - $F_{e\ cr}$ is the critical buckling load;

- $E = 2 \cdot 10^6 \text{ [daN/cm}^2\text{]}$ is the modulus of elasticity of the material;

- $I = \pi \cdot d^4 / 64 \text{ [cm}^4\text{]}$ represents the moment of inertia;

- l_f the buckling length, determined according to the cylinder clamping scheme.

The buckling length depends on the type of clamping elements of the rod

and the cylinder body and is determined according to the cylinder clamping scheme (fig. 7.5).

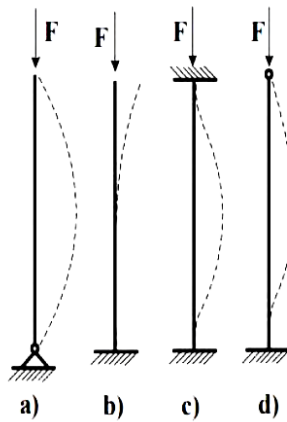


Fig.7.5. Bulking cases

The critical buckling length has the value:

- $l_f = l$, for case a;

- $l_f = 2 \cdot l$, for case b;

- $l_f = l/2$, for case c;

- $l_f = 0.7 \cdot l$, for case d;

where l is the length of the rod.

Considering that the cylinder is most often mounted articulated, the most used scheme for computing the buckling length for the piston rod is the one in figure 7.4, so that the equation for the critical buckling force becomes:

$$F_{cr} = 2.07 \cdot 10^7 \cdot \frac{d^4}{l^2} [N] \quad [1.2]$$

where: - F_{cr} is the critical buckling load;

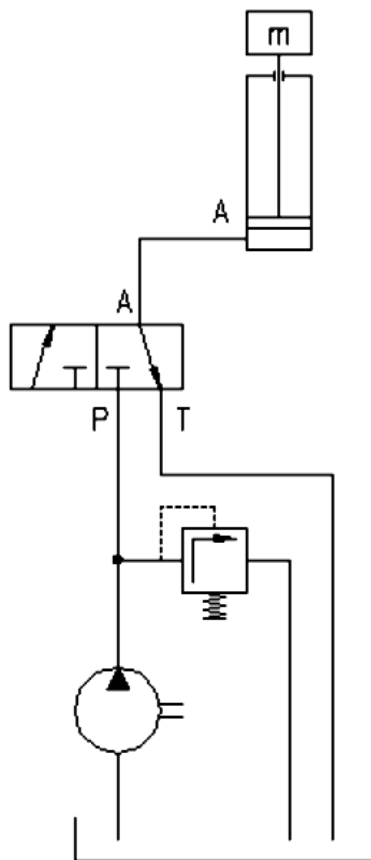
- d is the rod diameter in cm.

7.2. Purpose of the experiment

The paper aims to familiarize students with performing the necessary steps to size, verify or find the working parameters of hydraulic and pneumatic pystems.

7.3. Experimental procedure

- Measure the parameters of the piston and add them to Table 7.1;
- Create the following actiation diagram using the existing components in the laboratory and create the physical installation;



- Start the compressor and set the working parameters;

- Make reading for the pressure using the manometer of the installation;
- Find the time in which the piston does a full stroke and write the value in table 7.2;
- Repeat the procedure;
- Compute the force, speed, average speed and errors using the formulas provided in the laboratory.

Table 7.1. Cylinder dimensions

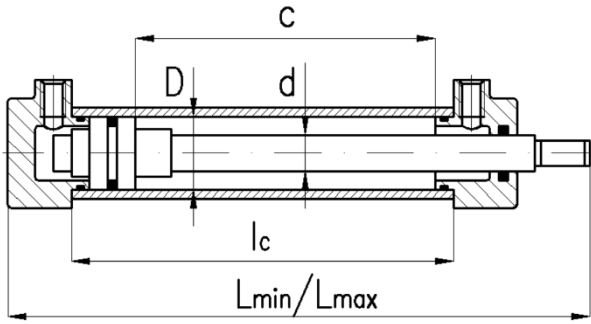
				
No.	Dimension	Symbol	MU	Measured Value
1	Piston diameter	D	mm	
2	Rod diameter	d	mm	
3	Piston stroke	c	mm	
4	Liner length	lc	mm	
5	Minimum cylinder length (retracted)	Lmin	mm	
6	Maximum cylinder length (extended)	Lmax	mm	

Table7.2. Experimental data

No.	Pressure [Pa]	Stroke [mm]	Time [s]	Force [N]	Speed [m/s]	Average speed [m/s]	Error [%]
1.							
2.							
3.							
4.							
5.							
6.							
7.							
8.							
9.							
10.							

8. NUMERICAL APPLICATIONS FOR PRACTICAL WORKS

Theoretical aspects

The formulas necessary to perform the calculations, for the proposed applications, are summarized in the following lines.

- **The thermal expansion** of the fluid can be expressed by the equation:

$$V_1 = V_0[1 + \alpha(t_1 - t_0)] \quad [8.1]$$

where V_1 is the volume of the liquid at temperature t_1 ;

V_0 is the initial volume of the liquid at temperature t_0 ;

α is the coefficient of thermal expansion [$1/K$], [$1/^\circ C$].

- **The compressibility of a fluid** is defined as a differential law of linear variation:

$$k = -\frac{1}{V_0} \cdot \frac{\Delta V}{\Delta p} \left[\frac{m^3}{N} \right] \quad [8.2]$$

where: k is the coefficient (modulus) of compressibility of the fluid [m^3/N];

V_0 is the initial volume of the fluid;

ΔV is the change in volume of the fluid;

Δp is the change in pressure acting on the fluid [N/m^2].

The equation can be written using the coefficient (modulus) of elasticity:

$$\varepsilon = \frac{1}{k} \left[\frac{N}{m^2} \right]; [Pa] \quad [8.3]$$

- **The heat flux density** Φ_q represents the heat flux that passes, from the hot surface to the cold surface, through the unit area of a surface with constant temperature and is given by Fourier's

equation:

$$\Phi_q = -k_q \cdot \frac{\Delta t}{h} \left[\frac{W}{m^2} \right] \quad [8.4]$$

where: Φ_q is the heat flux density [W/m^2];

Δt is the temperature difference between the cold layer and the hot layer of fluid [K], [$^{\circ}C$];

h is the distance between the cold and hot surfaces with which the fluid comes into contact [m];

k_q is the thermal conductivity coefficient of the fluid [W/mK].

The density represents the ratio between the mass and volume of the liquid and is determined by the equation:

$$\rho = \frac{m}{V} \left[\frac{kg}{m^3} \right] \quad [8.5]$$

where: ρ is the density of the fluid [kg/m^3];

m is the mass of the fluid [kg];

V is the volume of the fluid [m^3]

The density of a mixture of liquids is determined from the following equation:

$$\rho_{am} = \frac{\rho_1 \cdot V_1 + \rho_2 \cdot V_2 + \dots + \rho_n \cdot V_n}{V_1 + V_2 + \dots + V_n} \left[\frac{kg}{m^3} \right] \quad [8.6]$$

where: ρ_m is the density of the mixture [kg/m^3];

$\rho_1, \rho_2, \dots, \rho_n$ are the densities of the mixture components [kg/m^3];

V_1, V_2, \dots, V_n are the volumes of the mixture components [m^3].

Water presents an anomaly from this point of view, the maximum density being at a temperature of 3.98°C and has a value of 1000 kg/m³.

Relative density is the ratio between the density (volumetric mass) of a body at a temperature of $t^{\circ}\text{C}$ and the density (volumetric mass) of pure water at a temperature of $t_1^{\circ}\text{C}$, or in some cases the density (volumetric mass) of another reference substance at a temperature of $t_1^{\circ}\text{C}$ and is a dimensionless number symbolized d'_{t_1} .

Dynamic viscosity can be computed using the following equation:

$$\eta = \tau \cdot \frac{dn}{dv} [P] \quad [8.7]$$

where: η represents the dynamic viscosity [P];

τ is the tangential tension between two fluid layers;

dv/dn is the velocity gradient along the normal to the fluid flow direction.

The measurement unit for dynamic viscosity is Poise [P], with submultiple centiPoise [cP]:

$$1 \frac{N \cdot s}{m^2} = 10P = 10^3 cP \quad [8.8]$$

The kinematic viscosity is determined with the equation:

$$\nu = \frac{\eta}{\rho} \left[\frac{m^2}{s} \right] \quad [8.9]$$

where: ν is the kinematic viscosity of the fluid [St];

η is the dynamic viscosity of the fluid [P];

ρ is the density of the fluid [kg/m³].

The measurement unit for kinematic viscosity is Stokes [St] with a submultiple of centiStoke.

$$\begin{aligned} 1 \text{ St} &= 1 \frac{\text{cm}^2}{\text{s}} \\ 1 \text{ cSt} &= 1 \frac{\text{mm}^2}{\text{s}} \end{aligned} \quad [8.10]$$

Viscosity can also be expressed in degrees Engler [°E].

The number of Engler degrees is:

$$E = \frac{t}{t_0} \quad [8.11]$$

where: t is the time for the fluid volume to flow [s];

t_0 is the time for the same volume of distilled water to flow [s].

One Engler degree corresponds to a kinematic viscosity:

$$\nu = 10^{-6} \left[\frac{\text{m}^2}{\text{s}} \right] \quad [8.12]$$

The conversion from Engler degrees is done using an empirical formula:

$$\nu = \left[7.32 \cdot {}^\circ\text{E} - \left(\frac{6.31}{{}^\circ\text{E}} \right) \right] \cdot 10^{-6} \left[\frac{\text{m}^2}{\text{s}} \right] \quad [8.13]$$

The variation of kinematic viscosity with temperature for distilled water can be approximated by the following formula:

$$\nu_t = \frac{\nu_0}{1 + 0.0337 t + 0.00022 t^2} \left[\frac{\text{m}^2}{\text{s}} \right] \quad [8.14]$$

where: ν_t is the coefficient of kinematic viscosity of water at temperature t [m²/s];

$\nu_0 = 0,00000178 \text{ m}^2/\text{s}$ is the kinematic viscosity coefficient of water at a temperature of 0°C;

t is the temperature [$^{\circ}\text{C}$].

For petroleum products, the calculation of the variation of kinematic viscosity with temperature can be determined with the equation:

$$\lg(\lg(v_t + 0.7)) = A - B \cdot \lg(T) \quad [8.15]$$

where: v_t is the kinematic viscosity coefficient of the fluid in [mm^2/s] or [cSt] at the temperature T ;

A, B are numerical constants;

T is the temperature [K].

The kinematic viscosity of a mixture of liquids can be approximated with the formula:

$$v_m = \frac{v_1 \cdot \alpha_1 + v_2 \cdot \alpha_2 + \dots + v_n \cdot \alpha_n}{n} \left[\frac{\text{m}^2}{\text{s}} \right] \quad [8.16]$$

where: v_m is the kinematic viscosity of the mixture [m^2/s];

v_1, v_2, \dots, v_n are the viscosities of the mixture components [kg/m^3];

$\alpha_1, \alpha_2, \dots, \alpha_n$ are the volume percentages of the mixture components.

Surface tension can be expressed as:

$$\sigma = \frac{F}{l} \left[\frac{\text{N}}{\text{m}} \right] \quad [8.17]$$

where: σ is the surface tension coefficient [N/m], [din/cm];

F is the force generated by the surface tension [N];

l is the contour length of the surface layer [m];

The equivalence between the two units of measurement is:

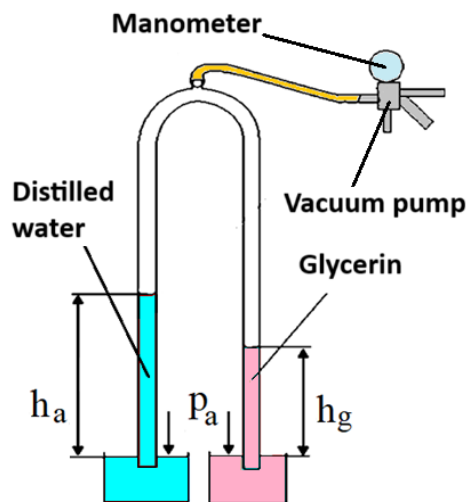
$$1 \frac{\text{din}}{\text{cm}} = 0.001 \frac{\text{N}}{\text{m}} \quad [8.19]$$

9. SOLVED EXAMPLES

Example 1

Hare's tube is an inverted U-shaped tube, with a connection socket at the top for a mini-vacuum pump (figure below) and is used to determine the density of liquids. In the two beakers, a reference liquid is placed, for which the density is known, and the liquid whose density is to be determined.

Both liquids are at a temperature of 200°C . After activating the vacuum pump and restoring the liquid level in the beakers, distilled water (the standard liquid) rises in the tube to a height of $h = 240\text{ mm}$. In the other tube, glycerin (the unknown liquid) rises to $h_g = 200\text{ mm}$. Knowing the density of water $\rho_a = 1000\text{ kg/m}^3$, and that the gravitational acceleration is $g = 9.81\text{ m/s}^2$, calculate the density ρ_g of glycerin.



Hare's Tube

For the tube filled in with distilled water we can write the following:

$$p_{atm} = p_{vid} + \rho_{water} \cdot g \cdot h_{water}$$

For the tube filled in with glycerin we can write the following:

$$p_{atm} = p_{vid} + \rho_g \cdot g \cdot h_g$$

If we consider the equality of the equations above we get:

$$p_{vid} + \rho_{water} \cdot g \cdot h_{water} = p_{vid} + \rho_g \cdot g \cdot h_g$$

The density of glycerin is:

$$\rho_g = \rho_{water} \cdot \frac{h_{water}}{h_g} = 1000 \cdot \frac{0.24}{0.2} = 1200 \frac{kg}{m^3}$$

Example 2

A mixture of ethyl alcohol and distilled water, at a temperature of 200°C, has a density of $\rho = 850 \text{ kg/m}^3$. Knowing that at a temperature of 200°C, ethyl alcohol has a density of $\rho_{alcohol} = 789.5 \text{ kg/m}^3$, and distilled water has a density of $\rho_{water} = 998.4 \text{ kg/m}^3$, calculate the volumetric concentration of ethyl alcohol in the mixture.

Solution

The density of a mixture of liquids is determined from the equation:

$$\rho = \frac{\rho_{alcohol} \cdot V_{alcohol} + \rho_{water} \cdot V_{water}}{V}$$

where: ρ is the density of the mixture;

$\rho_{alcohol}$ is the density of ethyl alcohol;

ρ_{water} is the density of distilled water;

$V_{alcohol}$ is the volume of ethyl alcohol in the mixture;

V_{water} is the volume of distilled water in the mixture;

V is the volume of the mixture of liquids expressed as:

$$V = V_{alcohol} + V_{water}$$

For the volume of distilled water we have:

$$V_{water} = V - V_{alcohol}$$

If we make the substitutions in the first formula we get:

$$\begin{aligned}\rho &= \frac{\rho_{alcohol} \cdot V_{alcohol} + \rho_{water} \cdot (V - V_{alcohol})}{V} \\ &= \frac{\rho_{alcohol} \cdot V_{alcohol} + \rho_{water} \cdot V - \rho_{water} \cdot V_{alcohol}}{V} \\ &= \frac{(\rho_{alcohol} - \rho_{water}) \cdot V_{alcohol}}{V} + \rho_{water}\end{aligned}$$

We get:

$$\frac{(\rho_{alcohol} - \rho_{water}) \cdot V_{alcohol}}{V} = \rho - \rho_{water}$$

The volumetric ratio is:

$$\frac{V_{alcohol}}{V} = \frac{(\rho - \rho_{water})}{(\rho_{alcohol} - \rho_{water})} = \frac{850 - 998.4}{789.5 - 998.4} = 0.71 = 71\%$$

Example 3

A tank contains a volume $V_1 = 2500$ liters of oil with density $\rho_1 = 850$ kg / m^3 and a volume $V_2 = 1500$ liters of oil with density $\rho_2 = 860$ kg / m^3 .

Compute the density of the mixture in the tank.

Solution

The density of the mixture is computed using the formula:

$$\rho = \frac{(\rho_1 V_1 + \rho_2 V_2)}{(V_1 + V_2)} = \frac{850 \cdot 2500 + 860 \cdot 1500}{2500 + 1500} = 853.75 \frac{kg}{m^3}$$

Example 4

If the kinematic viscosity of distilled water $\nu_0 = 1.78 \cdot 10^{-6} \text{ m}^2/\text{s}$ at a temperature of 0°C is known, using the empirical equation of variation of the kinematic viscosity of distilled water with temperature from the manual, find the kinematic viscosity ν_{60} at a temperature of 60°C .

Solution

In order to compute the kinematic viscosity we use the following equation:

$$\nu_t = \frac{\nu_0}{1 + 0,0337 t + 0,00022 t^2} \left[\frac{\text{m}^2}{\text{s}} \right]$$

where: $\nu_t [\text{m}^2/\text{s}]$ is the kinematic viscosity coefficient of water at temperature t ;

$\nu_0 = 1,78 \cdot 10^{-6} [\text{m}^2/\text{s}]$ is the coefficient of kinematic viscosity of water at a temperature of 0°C ;

$t [^\circ\text{C}]$ represents the temperature of the fluid.

The kinematic viscosity at 60°C is:

$$\nu_{60} = \frac{1.78 \cdot 10^{-6}}{1 + 0,0337 \cdot 60 + 0,00022 \cdot 60^2} = 0.466 \cdot 10^{-6} \left[\frac{\text{m}^2}{\text{s}} \right]$$

Example 5

The flow times for a volume of distilled water at 20°C , through the capillary of the Oswald viscometer, measured in seconds, to establish an average time, are: $t_1 = 342.98 \text{ s}$; $t_2 = 344.02 \text{ s}$ and $t_3 = 342.12 \text{ s}$. The flow time for the same volume of ethanol, also at 20°C , is $t_e = 520.22 \text{ seconds}$.

Determine the kinematic viscosity of ethanol ν_e if the viscosity of water at 20°C is known, $\nu_{\text{water}} = 1.0023 \cdot 10^{-6} \text{ m}^2 / \text{s}$.

Solution

The average flow time of distilled water is calculated with the formula:

$$t_m = \frac{t_1 + t_2 + t_3}{3}$$

where t_m represents the average time;

t_1, t_2, t_3 represent the time values determined for distilled water;

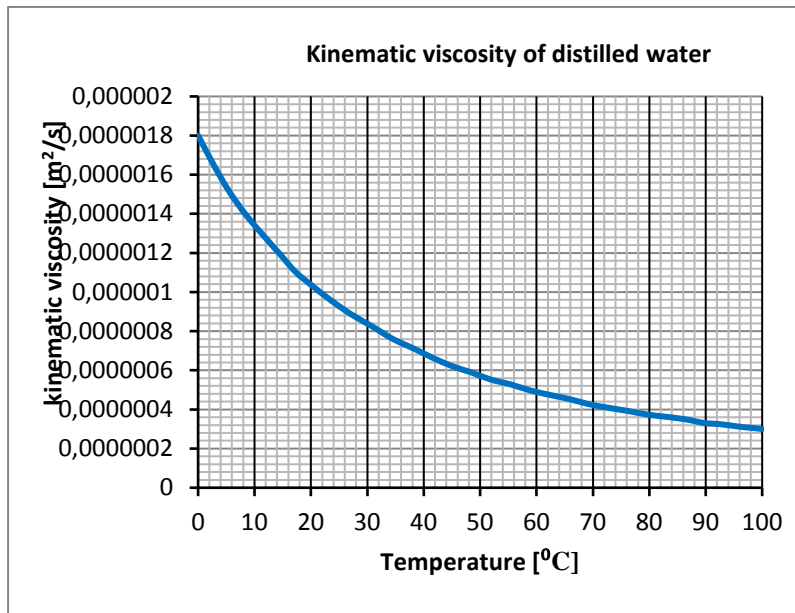
The constant of the device is determined by the formula:

$$k = \frac{\nu_w}{t_{med}}$$

where k is the device constant;

ν_w is the viscosity of distilled water, according to the graph in figure

below:



t_{med} is the average flow time of distilled water, for a given volume.

Substituting into the formula we have:

$$k = \frac{v_w}{t_{med}} = \frac{3 \cdot v_w}{t_1 + t_2 + t_3} = \frac{3 \cdot 1.0023 \cdot 10^{-6}}{342.98 + 344.02 + 342.12}$$

$$= 0.0029 \cdot 10^{-6} \left[\frac{m^2}{s^2} \right]$$

The kinematic viscosity of ethanol is calculated with the equation:

$$v_e = k \cdot t_e = 0.0029 \cdot 10^{-6} \cdot 520.22 = 1.519 \cdot 10^{-6} \left[\frac{m^2}{s} \right]$$

Example 6

Compute the average value of the dynamic viscosity of a liquid at a temperature of 25°C with the Brookfield rotational viscometer if only the rotating body with number 3 is used, with which the reading $\alpha_1 = 13$ is obtained, for the speed of the viscometer 12, the reading $\alpha_2 = 33$, for the speed of the viscometer 30 and the reading $\alpha_3 = 59$, for the speed of the viscometer 60. The viscometer constant is in the table below:

Speed level	Rotational bodies number			
	1	2	3	4
6	10	50	200	1000
12	5	25	100	500
30	2	10	40	200
60	1	5	20	100

Solution

The dynamic viscosity measured with the Brookfield apparatus is determined by the formula:

$$\eta = k_{i-j} \cdot \alpha$$

where: η [mPas]- dynamic viscosity;

k_{i-j} - coefficient depending on the number of rotating body (i) and speed (j), from table above.

α - reading on the device dial.

For each step we have:

$$\eta_1 = k_{3-12} \cdot \alpha_1 = 100 \cdot 13 = 1300 \text{ mPa} \cdot \text{s}$$

$$\eta_2 = k_{3-30} \cdot \alpha_2 = 40 \cdot 33 = 1320 \text{ mPa} \cdot \text{s}$$

$$\eta_3 = k_{3-60} \cdot \alpha_3 = 20 \cdot 59 = 1180 \text{ mPa} \cdot \text{s}$$

From which it follows that the average value of the dynamic viscosity is:

$$\eta = \frac{(\eta_1 + \eta_2 + \eta_3)}{3} = \frac{1300 + 1320 + 1180}{3} = 1266.67 [\text{mPas}]$$

Example 7

A stalagmometer has a capillary tube diameter $d = 2$ mm. The stalagmometer is charged with distilled water at a temperature of 25°C and a number $n = 100$ drops are allowed to drip into a glass. The water in the glass has a mass $m = 4.61\text{g}$. Knowing that the gravitational acceleration is $g = 9.81 \text{ m/s}^2$, calculate the surface tension of the distilled water.

Solution

The weight of a drop is:

$$G_p = \frac{m \cdot g}{n}$$

The surface tension force of a drop is:

$$F = \sigma \cdot \pi \cdot d$$

Equating the weight of a drop with the surface tension force, we obtain:

$$\sigma \cdot \pi \cdot d = \frac{m \cdot g}{n}$$

From where:

$$\sigma = \frac{m \cdot g}{n \cdot \pi \cdot d} = \frac{4.61 \cdot 10^{-3} \cdot 9.81}{100 \cdot \pi \cdot 2 \cdot 10^{-3}} = 0.07192 \frac{N}{m}$$

Example 8

For the relative method of determining the surface tension of a fluid, a standard liquid is used, whose surface tension is known. The stalagmometer (Traube tube) is charged with a volume of distilled water (standard liquid) and allowed to drip. For water, a number $n_0 = 100$ drops are obtained. The surface tension of distilled water $\sigma_0 = 71.97 \text{ mN / m}$ and the density $\rho_0 = 1 \text{ kg / dm}^3$ are known, at a temperature of 20°C . The same stalagmometer is charged with the same volume of glycerin and allowed to drip. For glycerin, a number $n = 141$ drops are obtained. Knowing that the density of glycerin is $\rho = 1.26 \text{ kg / dm}^3$, calculate the surface tension σ of glycerin.

Solution

Using the specific equation of this method we obtain:

$$\sigma = \frac{\rho \cdot n_0}{\rho_0 \cdot n} \cdot \sigma_0 = \frac{1.26 \cdot 100}{1 \cdot 140} \cdot 71.97 = 64.73 \left[\frac{mN}{m} \right]$$

Example 9

An Engler viscometer registers 21°E for the viscosity of a hydraulic oil. Knowing that the density of the oil is $\rho = 0.890 \text{ kg / dm}^3$, it is required to calculate the kinematic viscosity ν and the dynamic viscosity η .

Solution

The formula for kinematic viscosity is:

$$\begin{aligned}\nu &= \left[7.32 \cdot {}^0E - \left(\frac{6.31}{{}^0E} \right) \right] \cdot 10^{-6} = \left[7.32 \cdot 21 - \left(\frac{6.31}{21} \right) \right] \cdot 10^{-6} \\ &= 153.419 \cdot 10^{-6} \left[\frac{\text{m}^2}{\text{s}} \right]\end{aligned}$$

Dynamic viscosity is computed with the formula:

$$\eta = \rho \cdot \nu = 890 \cdot 153.419 \cdot 10^{-6} = 0.13654 \text{ Pa} \cdot \text{s}$$

Equation 10

A hydraulic fluid, of petroleum origin, has at a temperature of 15°C, a kinematic viscosity ν_t of 3.669 cSt (1cSt = 1mm²/s = 10⁻⁶m²/s), and at a temperature of 80°C, a kinematic viscosity ν_t of 1.336 cSt. Using the formula for calculating the variation of kinematic viscosity with temperature, for hydraulic fluids, from the manual, find the kinematic viscosity of this fluid at a temperature of 40°C.

Using table below ISO 3448 viscosity classes, at 40°C, from the manual, with the viscosity grades, establish the viscosity grade corresponding to this fluid.

Nr.	Viscosity cat. ISO	Average Kinematic Viscosity [mm ² /s]	Kinematic viscosity- limit values [mm ² /s]	
			Min.	Max.
1	ISO VG 2	2,2	1,98	2,42
2	ISO VG 3	3,2	2,88	3,52
3	ISO VG 5	4,6	4,14	5,06
4	ISO VG 7	6,8	6,12	7,48
5	ISO VG 10	10	9,00	11,00

Solution

The formula for the variation of viscosity with temperature, for hydraulic fluids, is:

$$\lg(\lg(v_t + 0.7)) = A - B \cdot \lg(T)$$

where: v_t is the coefficient of kinematic viscosity of the fluid in [mm²/s] or [cSt] at the temperature T;

A, B are numerical constants;

T represents the temperature in [K].

In the first step, the values of the two numerical constants A and B must be determined. For this, the given equation is written for the two temperatures and the two viscosities:

$$\lg(\lg(3.699 + 0.7)) = A - B \cdot \lg(15 + 273.15)$$

$$\lg(\lg(1.336 + 0.7)) = A - B \cdot \lg(80 + 273.15)$$

From the first equation we get:

$$A = \lg(\lg(3.699 + 0.7)) + B \cdot \lg(15 + 273.15)$$

From the second equation we get:

$$A = \lg(\lg(1.336 + 0.7)) + B \cdot \lg(80 + 273.15)$$

From the formulas above we get the values for A and B:

$$A = 8.621771$$

$$B = 3.584354$$

The equation takes the form:

$$\lg(\lg(v_t + 0.7)) = 8.621771 - 3.584354 \cdot \lg(T)$$

And for the temperature of 40°C, we get:

$$\lg(\lg(v_t + 0.7)) = 8.621771 - 3.584354 \cdot \lg(40 + 273.15)$$

Solving the equation we get:

$$\lg(\lg(v_t + 0.7)) = 0.32314$$

$$\lg(v_t + 0.7) = 10^{-0.32314} = 0.47518$$

$$v_t = 10^{-0.47518} - 0.7 = 2.98622 - 0.7 = 2.28622 \text{ cSt}$$

According to the table above the viscosity class of this hydraulic fluid is ISO VG 2 with a viscosity between 1.98 and 2.42 cSt (mm²/s).

Example 11

A tank filled with water at a temperature of $t_0 = 40^\circ\text{C}$ is sealed. The tank is heated to a temperature of $t = 80^\circ\text{C}$. Knowing that the coefficient of isobaric volumetric expansion of water is $\alpha = 1.8 \cdot 10^{-4} \text{ K}^{-1}$, and the coefficient of isothermal compressibility is $\beta = 4.19 \cdot 10^{-10} \text{ m}^2/\text{N}$, compute how much the pressure in the tank increases if the expansion of the tank is neglected.

Solution

Increasing the temperature of water normally produces an increase in volume, due to thermal expansion. The increase in volume would be:

$$\Delta V = V_0 \cdot \alpha \cdot (t - t_0)$$

where ΔV is the volumic variation of the water, due to heating;

V_0 is the initial volume of water in the tank;

α is the coefficient of isobaric volumetric expansion of water;

t is the final temperature of the water;

t_0 is the initial temperature of the water.

Because the tank does not deform, this volume variation produces a compression on the water, according to the formula:

$$\Delta = V_0 \cdot \beta \cdot \Delta p$$

where ΔV is the volumic variation of water produced by compression;

V_0 is the initial volume of water in the tank;

β is the isothermal compressibility coefficient of water;

Δp is the pressure variation due to the volumic variation ΔV .

Equating the two equations we obtain:

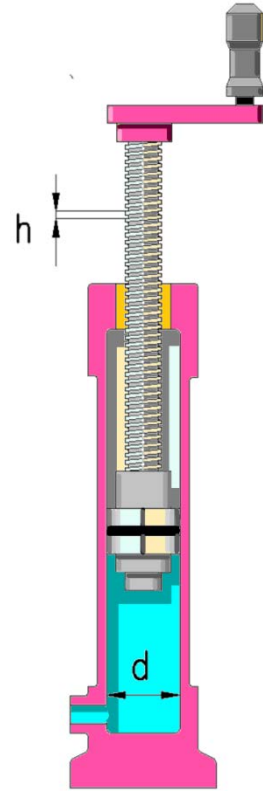
$$V_0 \cdot \alpha \cdot (t - t_0) = V_0 \cdot \beta \cdot \Delta p$$

From where we get:

$$\begin{aligned} \Delta p &= \frac{V_0 \cdot \alpha \cdot (t - t_0)}{V_0 \cdot \beta} = \frac{\alpha \cdot (t - t_0)}{\beta} = \frac{1.8 \cdot 10^{-4} \cdot (80 - 40)}{4.19 \cdot 10^{-10}} \\ &= 17183771 \text{ Pa} = 171.83771 \cdot 10^{-5} \text{ Pa} \end{aligned}$$

Example 12

A press has a cylinder piston of diameter $d = 2 \text{ cm}$ and is driven by a screw with pitch $h = 2 \text{ mm}$ as shown in the figure below. The cylinder has a volume of $V_0 = 0.2 \text{ liters}$ of hydraulic oil with isothermal compressibility coefficient $\beta = 4.8 \cdot 10^{-10} \text{ m}^2/\text{N}$ and has the connections blocked. What is the final pressure in the cylinder (p_f) if the initial pressure in the cylinder is $p_0 = 0 \text{ bar}$ and the cylinder operating lever is rotated $n = 2$ revolutions?



Hydraulic press

Solution

The variation in oil volume, following rotation of the actuating lever, is:

$$\Delta V = \frac{\pi \cdot d^2}{4} \cdot n \cdot h$$

From the formula for the compressibility modulus:

$$\Delta V = \beta \cdot V_0 \cdot \Delta p = \beta \cdot V_0 \cdot (p_f - p_i)$$

we have for the variation of the oil pressure in the cylinder:

$$(p_f - p_i) = \frac{\Delta V}{\beta \cdot V_0} = \frac{\pi \cdot d^2}{4} \cdot \frac{n \cdot h}{\beta \cdot V_0}$$

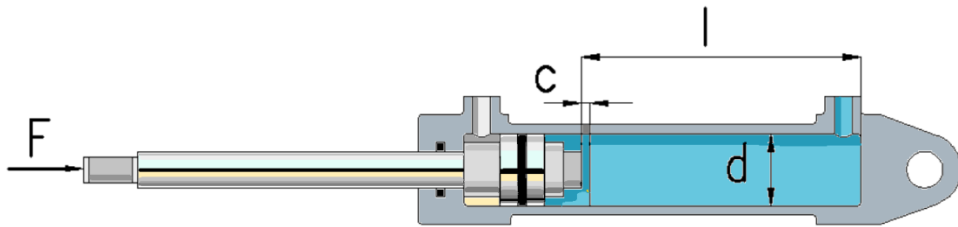
From the equation above we get:

$$p_f = \frac{\pi \cdot d^2}{4} \cdot \frac{n \cdot h}{\beta \cdot V_0} + p_i = \frac{\pi \cdot 0.02^2}{4} \cdot \frac{2 \cdot 0.002}{4.8 \cdot 10^{-10} \cdot 0.2 \cdot 10^{-3}} + 0$$

$$= 13089969 \frac{N}{m^2} = 130.89 \text{ bar}$$

Example 13

The diameter of the piston of a cylinder is $d = 30 \text{ mm}$. The cylinder has sealed connections and is filled with hydraulic oil with isothermal compressibility coefficient $\beta = 4.8 \cdot 10^{-10} \text{ m}^2/\text{N}$ (figure below). The inner length of the cylinder is $l = 800 \text{ mm}$. If the pressure in the cylinder is $p_0 = 0 \text{ bar}$, with what force must the cylinder rod be pressed so that it moves with the stroke $c = 2 \text{ mm}$, compressing the oil in the cylinder?



Hydraulic cylinder

Solution

The initial volume of oil in the cylinder is:

$$V_0 = \frac{\pi \cdot d^2}{4} \cdot l$$

The final volume of oil in the cylinder is:

$$V_f = \frac{\pi \cdot d^2}{4} \cdot (l - c)$$

The variation in the volume of oil in the cylinder is:

$$\Delta V = V_0 - V_f$$

From the equation for the compressibility modulus:

$$\Delta V = \beta \cdot V_0 \cdot \Delta p = \beta \cdot V_0 \cdot (p_f - p_0)$$

we have for the variation of the oil pressure in the cylinder:

$$(p_f - p_0) = \frac{\Delta V}{\beta \cdot V_0} = \frac{V_0 - V_f}{\beta \cdot V_0} = \frac{\pi \cdot d^2}{4} \frac{c}{\beta \cdot \frac{\pi \cdot d^2}{4} \cdot l} = \frac{c}{\beta \cdot l}$$

The final pressure in the cylinder will have the value:

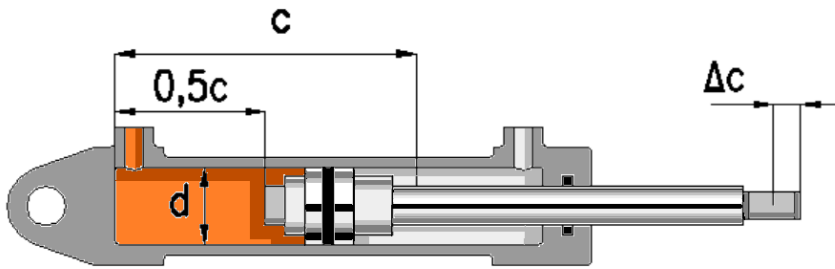
$$p_f = \frac{c}{\beta \cdot l} + p_0$$

The force in the cylinder rod will be:

$$F = \frac{\pi \cdot d^2}{4} \cdot p_f = \frac{\pi \cdot d^2}{4} \cdot \left(\frac{c}{\beta \cdot l} + p_0 \right) = \frac{\pi \cdot 0.03^2}{4} \cdot \left(\frac{0.002}{4.8 \cdot 10^{-10} \cdot 0.8} + 0 \right) \\ = 3681.55 \text{ N}$$

Example 14

Compute the stroke Δc that the piston of a cylinder filled with hydraulic oil with an isobaric volumetric expansion coefficient $\alpha = 7 \cdot 10^{-4} \text{ K}^{-1}$ will make, when the oil temperature increases by $\Delta t = 100^\circ\text{C}$, and the oil in the cylinder expands. The maximum stroke of the cylinder piston is $c = 300\text{mm}$, and the piston is at half its stroke. It is assumed that the oil is incompressible, and the cylinder does not expand with increasing temperature.



Solution

The initial volume of oil in the cylinder is:

$$V_0 = \frac{\pi \cdot d^2}{4} \cdot \frac{c}{2} = \frac{\pi \cdot d^2 \cdot c}{8}$$

where V_0 is the initial volume of oil in the cylinder;

d is the diameter of the piston;

c is the piston stroke.

The increase in volume due to heating of the oil is:

$$\Delta V = V_0 \cdot \alpha \cdot \Delta t$$

where ΔV is the volume variation of the oil in the cylinder;

V_0 is the initial volume of oil in the cylinder;

α is the coefficient of isobaric volumetric expansion of the oil;

Δt is the temperature variation of the oil in the cylinder.

Substituting V_0 in the first formula, we obtain for the volume variation:

$$\Delta V = \frac{\pi \cdot d^2 \cdot c \cdot \alpha \cdot \Delta t}{8}$$

The volume variation ΔV produces the stroke variation Δc , according to the formula:

$$\Delta V = \frac{\pi \cdot d^2}{4} \cdot \Delta c$$

where ΔV is the volume variation of the oil in the cylinder;

d is the piston diameter;

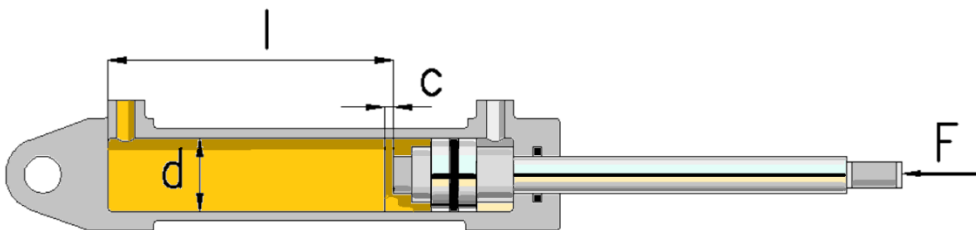
Δc is the variation in piston stroke corresponding to the variation in volume ΔV .

We have:

$$\Delta c = \frac{4 \cdot \Delta V}{\pi \cdot d^2} = \frac{4 \cdot \frac{\pi \cdot d^2 \cdot c \cdot \alpha \cdot \Delta t}{8}}{\pi \cdot d^2} = \frac{c \cdot \alpha \cdot \Delta t}{2} = \frac{0.3 \cdot 7 \cdot 10^{-4} \cdot 100}{2} = 0.0105 \text{ mm}$$

Example 15

A piston cylinder contains hydraulic oil with an isobaric volumetric expansion coefficient $\alpha = 7 \cdot 10^{-4} \text{ K}^{-1}$ and an isothermal compressibility coefficient $\beta = 4.8 \cdot 10^{-10} \text{ m}^2/\text{N}$. The piston diameter is $d = 25 \text{ mm}$. Calculate the variation ΔF by which the force applied to the piston rod must increase in order for it not to move when the oil temperature increases by $\Delta t = 40^\circ\text{C}$. It is assumed that the cylinder does not expand with increasing temperature.



Solution

The initial volume of oil in the cylinder is:

$$V_0 = \frac{\pi \cdot d^2}{4} \cdot c$$

where V_0 is the initial volume of oil in the cylinder;

d is the diameter of the piston;

c is the piston stroke.

The increase in volume due to heating of the oil is:

$$\Delta V = V_0 \cdot \alpha \cdot \Delta t$$

where ΔV is the volume variation of the oil in the cylinder;

V_0 is the initial volume of oil in the cylinder;

α is the coefficient of isobaric volumetric expansion of the oil;

Δt is the temperature variation of the oil in the cylinder.

Substituting V_0 in the first formula, we obtain for the volume variation:

$$\Delta V = \frac{\pi \cdot d^2 \cdot c \cdot \alpha \cdot \Delta t}{4}$$

Because the piston does not move, this volume variation, ΔV , produces a compression on the oil, according to the formula:

$$\Delta V = \beta \cdot V_0 \cdot \Delta p = \frac{\pi \cdot d^2 \cdot c \cdot \beta \cdot \Delta p}{4}$$

where ΔV is the volumic variation of water produced by compression;

V_0 is the initial volume of water in the tank;

β is the isothermal compressibility coefficient of water;

Δp is the pressure variation due to the volumic variation ΔV .

From the equations above we get:

$$\frac{\pi \cdot d^2 \cdot c \cdot \alpha \cdot \Delta t}{4} = \frac{\pi \cdot d^2 \cdot c \cdot \beta \cdot \Delta p}{4}$$

$$\alpha \cdot \Delta t = \beta \cdot \Delta p$$

The variation of pressure is:

$$\Delta p = \alpha \cdot \frac{\Delta t}{\beta}$$

And the force variation:

$$\Delta F = \Delta p \cdot \frac{\pi \cdot d^2}{4}$$

where ΔF is the variation of the force applied to the piston;

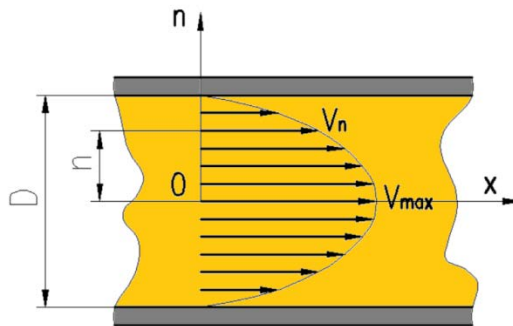
d is the piston diameter.

Replacing Δp we get:

$$\Delta F = \alpha \cdot \frac{\Delta t}{\beta} \cdot \frac{\pi \cdot d^2}{4} = 7 \cdot 10^{-4} \cdot \frac{40}{4.8 \cdot 10^{-10}} \cdot \frac{\pi \cdot 0.025^2}{4} = 28634.30 [N]$$

Example 16

Determine the minimum and maximum value of the tangential stress, for an oil with dynamic viscosity $\eta = 8.60 \cdot 10^{-1} \text{ Pa} \cdot \text{s}$ circulating through a pipe of diameter $D = 10 \text{ cm}$. The law of velocity variation is known, $v_n = v_{\max} (1 - 200 n^2 / D)$, where n is the radius corresponding to the layer with velocity v_n , according to figure. The maximum velocity $v_{\max} = 0.3 \text{ m/s}$.



Oil velocity distribution in the pipeline

Solution

The tangential stress between two oil layers is proportional to the velocity gradient normal to the oil flow direction. The computation formula is:

$$\tau = -\eta \cdot \frac{dv}{dn}$$

Substituting the speed we get:

$$\begin{aligned}\tau &= -\eta \cdot \frac{d \left[v_{max} \left(1 - 200 \cdot \frac{n^2}{D} \right) \right]}{dn} = -\eta \cdot \frac{-200 \cdot v_{max} \cdot 2 \cdot n}{D} \\ &= 400 \cdot \eta \cdot v_{max} \cdot \frac{n}{D}\end{aligned}$$

The minimum stress is obtained for $n = 0$:

$$\tau_{min} = 400 \cdot \eta \cdot v_{max} \cdot \frac{n}{D} = 400 \cdot 8.6 \cdot 10^{-1} \cdot 0.3 \cdot \frac{0}{0.10} = 0$$

The maximum stress is obtained for $n = D / 2 = 0.10 / 2 = 0.05$:

$$\tau_{max} = 400 \cdot \eta \cdot v_{max} \cdot \frac{n}{D} = 400 \cdot 8.6 \cdot 10^{-1} \cdot 0.3 \cdot \frac{0.05}{0.10} = 51.6 \text{ [Pa]}$$

Example 17

The law of variation of the velocity of a liquid flowing through a pipe with diameter $D = 12 \text{ mm}$ is given by the relation $v_n = 1 - 5000 \cdot n^2 \text{ m / s}$, where n is the radius corresponding to the layer with velocity v_n . Determine its dynamic viscosity η knowing that the tangential tension at the wall is $\tau = 3.2 \text{ Pa}$.

Solution

The tangential stress between the oil layers is given by the relationship:

$$\tau = -\eta \cdot \frac{dv}{dn}$$

By replacing the speed we get:

$$\tau = -\eta \cdot \frac{d \left[v_{max} \left(1 - 5000 \cdot \frac{n^2}{D} \right) \right]}{dn} = -\eta \cdot (0 - 5000 \cdot 2 \cdot n)$$

$$= 10000 \cdot \eta \cdot n$$

From here we get:

$$\eta = \frac{\tau}{10000 \cdot n}$$

At the wall of the pipe we have $n = D / 2$ and therefore the equation becomes:

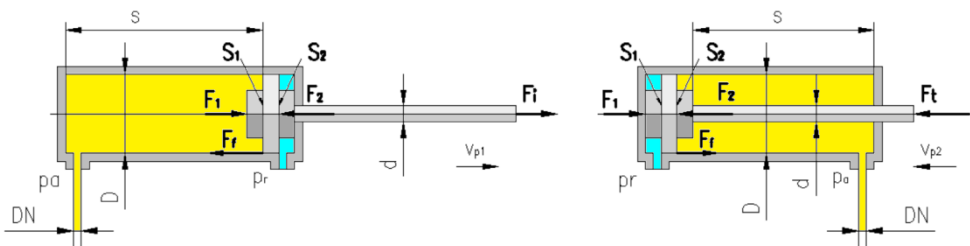
$$\eta = \frac{\tau}{10000 \cdot \frac{D}{2}}$$

By replacing we get:

$$\eta = \frac{\tau}{10000 \cdot \frac{D}{2}} = \frac{3.2}{10000 \cdot \frac{0.012}{2}} = 0.0533 \text{ [Pa} \cdot \text{s]}$$

Example 18

Find the thrust force F_1 and the pulling force F_2 of a double-acting cylinder that is supplied with hydraulic fluid at a pressure of $p = 25$ bar (figure below). The following are known: piston diameter $D = 80$ mm, rod diameter $d = 30$ mm, maximum piston stroke $s = 400$ mm. Neglect friction in the cylinder, the return pressure is zero.



Solution

For the calculation, dimensions are in meters, surfaces in m^2 , volumes in m^3 , speeds in m/s , pressures in Pascals, forces in N, and times in seconds.

Thrust force is the force generated on the left surface of the piston by the actuating pressure:

$$F_i = F_1 = p_a \cdot S_1 = p_a \cdot \pi \cdot \frac{D^2}{4} = 25 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2}{4} = 12566,37 \text{ N}$$

The pulling force is the force generated on the right surface of the piston by the actuating pressure:

$$\begin{aligned} F_t = F_2 = p_a \cdot S_2 &= p_a \cdot \pi \cdot \frac{D^2 - d^2}{4} = 25 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2 - 0,03^2}{4} \\ &= 10799,22 \text{ N} \end{aligned}$$

Example 19

Compute the thrust force F_i and the pulling force F_t of the same double-acting cylinder that is supplied with hydraulic fluid at a pressure of $p_a = 25$ bar, and the pressure on the return circuit is $p_r = 2$ bar. Disregard friction in the cylinder.

Solution

When there is back pressure, the thrust force is the difference between the force generated by the actuating pressure on the left surface and the force generated by the back pressure on the right surface of the piston:

$$\begin{aligned}
 F_i = F_1 - F_2 &= p_a \cdot S_1 - p_r \cdot S_2 = p_a \cdot \pi \cdot \frac{D^2}{4} - p_r \cdot \pi \cdot \frac{D^2 - d^2}{4} \\
 &= 25 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2}{4} - 2 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2 - 0,03^2}{4} \\
 &= 11702,43 \text{ N}
 \end{aligned}$$

The pulling force is the difference between the force generated by the actuating pressure on the right surface and the force generated by the return pressure on the left surface of the piston:

$$\begin{aligned}
 F_t = F_2 - F_1 &= p_a \cdot S_2 - p_r \cdot S_1 = p_a \cdot \pi \cdot \frac{D^2 - d^2}{4} - p_r \cdot \pi \cdot \frac{D^2}{4} \\
 &= 25 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2 - 0,03^2}{4} - 2 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2}{4} \\
 &= 9793,91 \text{ N}
 \end{aligned}$$

Example 20

Find the pushing force F_i and the pulling force F_t of the double-acting cylinder in the figure from example 18, which is supplied with hydraulic fluid at a pressure of $p_a = 25$ bar, and the pressure on the return circuit is $p_r = 2$ bar. We know: the piston diameter $D = 80$ mm, the rod diameter $d = 30$ mm, the maximum piston stroke $s = 400$ mm. The friction force in the cylinder is 20 percent of the force generated by the pressure in the cylinder.

Solution

The thrust force is the difference between the force generated by the actuating pressure on the left surface, the force generated by the return

pressure on the right surface of the piston, and the friction force, which is a percentage k of the force generated by the pressure p_a ($F_f = k \cdot F_1$):

$$\begin{aligned}
 F_i &= F_1 - F_2 - F_f = F_1 - F_2 - k \cdot F_1 = (1 - k) \cdot F_1 - F_2 \\
 &= (1 - k) \cdot p_a \cdot S_1 - p_r \cdot S_2 \\
 &= (1 - k) \cdot p_a \cdot \pi \cdot \frac{D^2}{4} - p_r \cdot \pi \cdot \frac{D^2 - d^2}{4} \\
 &= (1 - k) \cdot 25 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2}{4} - 2 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2 - 0,03^2}{4} \\
 &= 9189,15 \text{ N}
 \end{aligned}$$

The pulling force is the difference between the force generated by the actuating pressure on the right surface, the force generated by the return pressure on the left surface of the piston, and the friction force, which is a percentage k of the force generated by the pressure p_a ($F_f = k \cdot F_2$):

$$\begin{aligned}
 F_t &= F_2 - F_1 - F_f = F_2 - F_1 - k \cdot F_2 = (1 - k) \cdot F_2 - F_1 \\
 &= (1 - k) \cdot p_a \cdot S_2 - p_r \cdot S_1 \\
 &= (1 - k) \cdot p_a \cdot \pi \cdot \frac{D^2 - d^2}{4} - p_r \cdot \pi \cdot \frac{D^2}{4} \\
 &= (1 - k) \cdot 25 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2 - 0,03^2}{4} - 2 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2}{4} \\
 &= 7634,07 \text{ N}
 \end{aligned}$$

Example 21

For the cylinder in the figure from example 18, calculate the pressure p_1 and p_2 for which the cylinder develops the same force $F = 5000 \text{ N}$, both on the push and on the retraction. We know: the piston diameter $D = 80 \text{ mm}$, the rod diameter $d = 30 \text{ mm}$, the pressure on the return circuit is $p_r = 2$

bar. The friction force in the cylinder is 20 percent of the force generated by the pressure in the cylinder.

Solution

The thrust generated by the cylinder when supplied with pressure p_1 is:

$$F_i = F_1 - F_2 - F_f = F_1 - F_2 - k \cdot F_1 = (1 - k) \cdot F_1 - F_2 = (1 - k) \cdot p_1 \cdot S_1 - p_r \cdot S_2$$

From this equation we deduce the value of the pressure p_1 for a value of the force F .

$$\begin{aligned} p_1 &= \frac{F_i + p_r \cdot S_2}{(1 - k) \cdot S_1} = \frac{F + p_r \cdot \pi \cdot \frac{D^2 - d^2}{4}}{(1 - k) \cdot \pi \cdot \frac{D^2}{4}} \\ &= \frac{5000 + 2 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2 - 0,03^2}{4}}{(1 - 0,2) \cdot \pi \cdot \frac{0,08^2}{4}} \\ &= 1458241,74 \text{ Pa} \end{aligned}$$

The pulling force generated by the cylinder when supplied with pressure p_2 is:

$$F_t = F_2 - F_1 - F_f = F_2 - F_1 - k \cdot F_2 = (1 - k) \cdot F_2 - F_1 = (1 - k) \cdot p_2 \cdot S_2 - p_r \cdot S_1$$

From this equation we deduce the value of the pressure p_1 for a value of the force F .

$$\begin{aligned} p_2 &= \frac{F_t + p_r \cdot S_1}{(1 - k) \cdot S_2} = \frac{F + p_r \cdot \pi \cdot \frac{D^2}{4}}{(1 - k) \cdot \pi \cdot \frac{D^2 - d^2}{4}} \\ &= \frac{5000 + 2 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2}{4}}{(1 - 0,2) \cdot \pi \cdot \frac{0,08^2 - 0,03^2}{4}} = 1737772,21 \text{ Pa} \end{aligned}$$

Example 22

For the cylinder in the figure from example 18, calculate the volumes of liquid necessary to move the piston from one end to the other, on expansion and on retraction, respectively, the piston stroke being $s = 400$ mm.

Solution

The volume of liquid required to move the piston from left to right is the product of the left surface area of the piston and its stroke:

$$V_1 = s \cdot S_1 = s \cdot \pi \cdot \frac{D^2}{4} = 0,4 \cdot \pi \cdot \frac{0,08^2}{4} = 0,00201 \text{ m}^3$$

The volume of liquid required to move the piston from right to left is the product of the right surface area of the piston and its stroke:

$$\begin{aligned} V_2 &= s \cdot S_2 = s \cdot \pi \cdot \frac{D^2 - d^2}{4} = 0,4 \cdot \pi \cdot \frac{0,08^2 - 0,03^2}{4} \\ &= 0,00172 \text{ m}^3 \end{aligned}$$

Example 23

For the cylinder in the figure from example 18, calculate the pushing speed and the retraction speed of the piston if the hydraulic cylinder is powered by a pump with a volumetric flow rate $Q = 0.2 \text{ l/s}$. Neglect losses.

Solution

The speed of the piston moving to the left is proportional to the feed rate and inversely proportional to the left section of the piston. The thrust speed is:

$$v_i = \frac{Q}{S_1} = \frac{Q}{\pi \cdot \frac{D^2}{4}} = \frac{0,2 \cdot 10^{-3}}{\pi \cdot \frac{0,08^2}{4}} = 0,039 \text{ m/s}$$

The speed of the piston moving to the right is proportional to the feed rate and inversely proportional to the right section of the piston. The draw speed is:

$$v_t = \frac{Q}{S_2} = \frac{Q}{\pi \cdot \frac{D^2 - d^2}{4}} = \frac{0,2 \cdot 10^{-3}}{\pi \cdot \frac{0,08^2 - 0,03^2}{4}} = 0,046 \text{ m/s}$$

Example 24

Find the volume corresponding to a certain rotation of the pump (q) so that it ensures a flow rate $Q = 0.2 \text{ l/s}$ at a speed $n = 1300 \text{ rpm}$.

Solution

The volume corresponding to one pump rotation (q) is calculated with the formula:

$$q = \frac{60 \cdot Q}{n} = \frac{60 \cdot 0,2 \cdot 10^{-3}}{1300} = 9,230 \cdot 10^{-6} \text{ m}^3/\text{rot}$$

Example 25

If the pump has a volume corresponding to a certain rotation $q = 15 \text{ cm}^3/\text{rotation}$, calculate the pump drive speed so that the cylinder thrust speed is $v_1 = 0.1 \text{ m/s}$. Neglect losses.

Solution

The pump drive speed for the cylinder thrust speed to be $v_1 = 0.1$ m/s, when losses are neglected, is obtained by equating the pump flow rate with the flow rate necessary to drive the piston:

$$Q_p = \frac{q \cdot n}{60} = Q_c = v_1 \cdot S_1$$

From this equality we obtain:

$$\begin{aligned} n &= \frac{60 \cdot v_1 \cdot S_1}{q} = \frac{60 \cdot v_1 \cdot \pi \cdot \frac{D^2}{4}}{q} = \frac{60 \cdot 0,1 \cdot \pi \cdot \frac{0,08^2}{4}}{15 \cdot 10^{-6}} \\ &= 2010,61 \text{ rot/min} \end{aligned}$$

Example 26

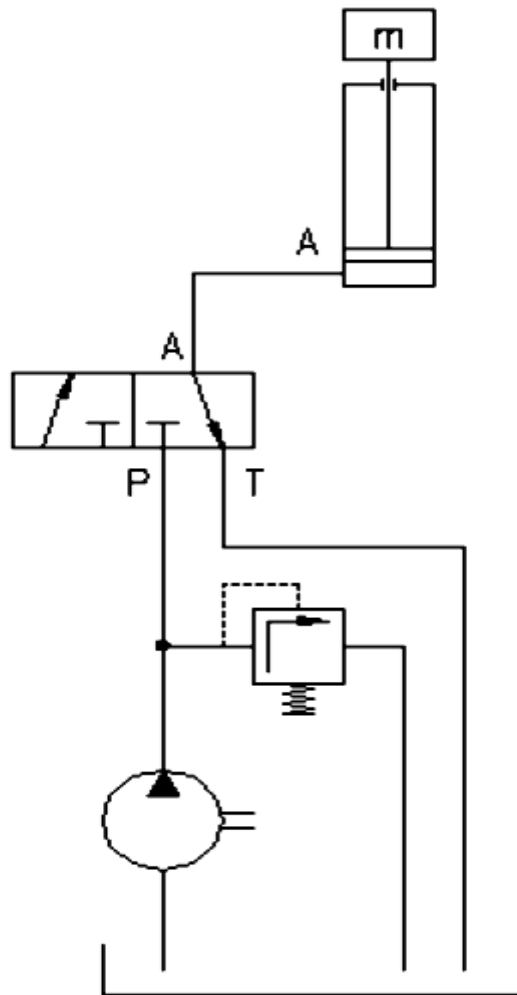
For the installation in the figure below, which must lift a load, the following are known:

- The load has a mass of 2500 kg.
- The pneumatic cylinder has a piston diameter $D = 60$ mm, a rod diameter $d = 20$ mm, a piston stroke $s = 600$ mm. The friction force in the cylinder is $F_f = 0.1 F_{\text{piston}}$.
- The pressure drop across the hydraulic distributor is $p_d = 4$ bar.
- The pump is driven at a speed $n = 1200$ rpm.
- The volume displaced by the pump corresponding to a single rotation $q = 0.02$ l/rotation.
- The internal diameter of the pipes is $D_n = 10$ mm.
- The maximum speed of the hydraulic fluid through the pipes is $v_u = 7$ m/s. The pressure drop across the pipe is neglected.

- The safety valve is set at $p_s = 130$ bar.

For this installation find the following:

- The pressure in the cylinder at the moment of lifting the load. Neglect inertial forces.
- The pressure that the pump must develop during lifting.
- The velocity of the fluid through the pipe during lifting the load.
- The speed with which the cylinder lifts the load.
- The speed with which the load lowers.
- The time to lift the load.
- The time to lower the load.
- The maximum mass that this installation can lift



Solution

The force that the cylinder must develop must overcome the load and the friction force, when inertia forces are neglected:

$$F_p = G + F_f = m \cdot g + k \cdot F_p$$

From where we get:

$$\begin{aligned} m \cdot g &= (1 - k) \cdot F_p = (1 - k) \cdot p_c \cdot S = (1 - k) \cdot p_c \cdot \pi \cdot \frac{D^2}{4} \\ &= 25 \cdot 10^5 \cdot \pi \cdot \frac{0,08^2}{4} \end{aligned}$$

And the pressure in the cylinder at the moment of lifting the load is:

$$p_c = \frac{m \cdot g}{(1 - k) \cdot S} = \frac{m \cdot g}{(1 - k) \cdot \pi \cdot \frac{D^2}{4}} = \frac{2500 \cdot 9,81}{(1 - 0,1) \cdot \pi \cdot \frac{0,06^2}{4}} = 9637716 \text{ Pa}$$

The pump must provide the pressure in the cylinder plus the pressure drop across the distributor. The pressure in the pump is:

$$p_p = p_c + p_d = 9637716 + 4 \cdot 10^5 = 10037716 \text{ Pa}$$

The velocity of the fluid in the pipe during the lifting of the load is:

$$v_f = \frac{Q_p}{s_c} = \frac{\frac{q \cdot n}{60}}{\pi \cdot \frac{DN^2}{4}} = \frac{\frac{0,02 \cdot 10^{-6} \cdot 1200}{60}}{\pi \cdot \frac{0,01^2}{4}} = 5,093 \text{ m/s}$$

The piston lifting speed is given by the feed flow rate.

$$v_r = \frac{Q_p}{S} = \frac{\frac{q \cdot n}{60}}{\pi \cdot \frac{D^2}{4}} = \frac{\frac{0,02 \cdot 10^{-6} \cdot 1200}{60}}{\pi \cdot \frac{0,06^2}{4}} = 0,141 \text{ m/s}$$

The piston's descent speed is limited to the maximum oil flow rate through the pipe.

$$v_c \cdot S = v_u \cdot s_c$$

It results for the piston descent speed:

$$v_c = \frac{v_u \cdot s_c}{S} = \frac{v_u \cdot \pi \cdot \frac{DN^2}{4}}{\pi \cdot \frac{D^2}{4}} = \frac{v_u \cdot DN^2}{D^2} = \frac{7 \cdot 0,01^2}{0,06^2} = 0,194 \text{ m/s}$$

The piston rise time is:

$$t_u = \frac{s}{v_r} = \frac{0,6}{0,141} = 4,241 \text{ s}$$

The piston descent time is:

$$t_c = \frac{s}{v_c} = \frac{0,6}{0,1410,194} = 3,086 \text{ s}$$

The maximum pressure that can reach the cylinder is equal to the safety valve pressure minus the pressure drop across the distributor:

$$p_{max} = p_s - p_d = 130 \cdot 10^5 - 4 \cdot 10^5 = 126 \cdot 10^5 \text{ Pa}$$

The maximum force that the cylinder generates is:

$$F_{max} = F_p - F_f = F_p - k \cdot F_p = (1 - k) \cdot F_p = m_{max} \cdot g$$

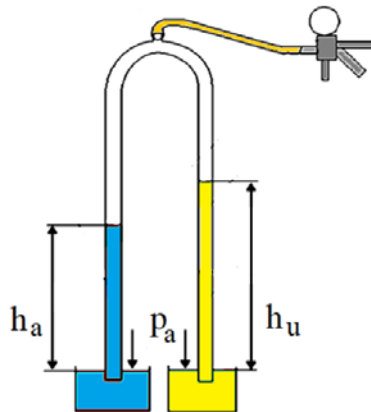
The maximum load the cylinder can lift is:

$$\begin{aligned} m_{max} &= \frac{(1 - k) \cdot F_p}{g} = \frac{(1 - k) \cdot p_{max} \cdot S}{g} = \frac{(1 - k) \cdot p_{max} \cdot \pi \cdot \frac{D^2}{4}}{g} \\ &= \frac{(1 - 0,1) \cdot 126 \cdot 10^5 \cdot \pi \cdot \frac{0,06^2}{4}}{9,81} = 3268,409 \text{ kg} \end{aligned}$$

10. PROPOSED EXAMPLES FOR SOLVING

Example 1

Using the Hare tube method, the density of a hydraulic oil is determined at a temperature of 20°C , using distilled water as a standard. After activating the vacuum pump and restoring the liquid level in the glasses, the distilled water (standard liquid) rises in the tube to the height $h_w = 140 \text{ mm}$, and in the other tube, the hydraulic oil rises to $h_{\text{oil}} = 159 \text{ mm}$. Knowing the density of water $\rho_a = 1000 \text{ kg/m}^3$, calculate the density ρ_{oil} of the hydraulic oil.



Example 2

A Hare tube contains distilled water and brake fluid at a temperature of 20°C . After activating the vacuum pump and restoring the liquid level in the glasses, the distilled water (standard liquid) rises in the tube to a height of 110 mm . Knowing the density of water $\rho_a = 1000 \text{ kg/m}^3$, the density of brake fluid $\rho_f = 1.060 \text{ kg/dm}^3$, compute the height h_f to which

the brake fluid rises.

Example 3

Find the density ρ of a mixture of 30% volume ethyl alcohol and 70% distilled water, knowing that at a temperature of 20°C ethyl alcohol has a density $\rho_{\text{alcohol}} = 789.5 \text{ kg / m}^3$, and distilled water has a density $\rho_w = 998,4 \text{ kg / m}^3$.

Example 4

A tank contains a mixture of 1.3 m³ of oil with a density of 0.843 g/cm³, 3500 liters of oil with a density of 0.862 kg/dm³, and 1200 dm³ of oil with a density of 892 kg/m³. Find the density of the mixture in the tank.

Example 5

In a tank containing a volume of 4 m³ of oil with a density of 0.830 g / cm³, 2000 liters of oil with an unknown density are poured. Knowing that the density of the mixture in the tank is 854 kg / m³, compute the density of the added oil.

Example 6

In two liters of hydraulic oil, with density $\rho_{\text{oil}} = 850 \text{ kg / m}^3$, 100 grams of additive, whose density is $\rho_{\text{additive}} = 1.3 \text{ kg / dm}^3$, are added. Determine the density of the additived oil ρ_{ao} .

Example 7

In what volumetric proportions should two liquids with densities $\rho_1 = 790$

kg / m^3 and $\rho_2 = 900 \text{ kg} / \text{m}^3$ be mixed, respectively, so that the resulting liquid has a density of $\rho = 0,890 \text{ kg} / \text{dm}^3$?

Example 8

Using the graphs of variation of water density with temperature, compute the volume of one kilogram of water for temperatures of 4°C , 10°C , 20°C , 40°C , 60°C , 80°C , 100°C .

Example 9

If the kinematic viscosity of distilled water $\nu_0 = 1.78 \cdot 10^{-6} \text{ m}^2/\text{s}$ at a temperature of 0°C is known, using the empirical relationship of variation of the kinematic viscosity of distilled water with temperature, from the manual, calculate the kinematic viscosity ν_{43} at a temperature of 43°C and ν_{78} at a temperature of 78°C .

Example 10

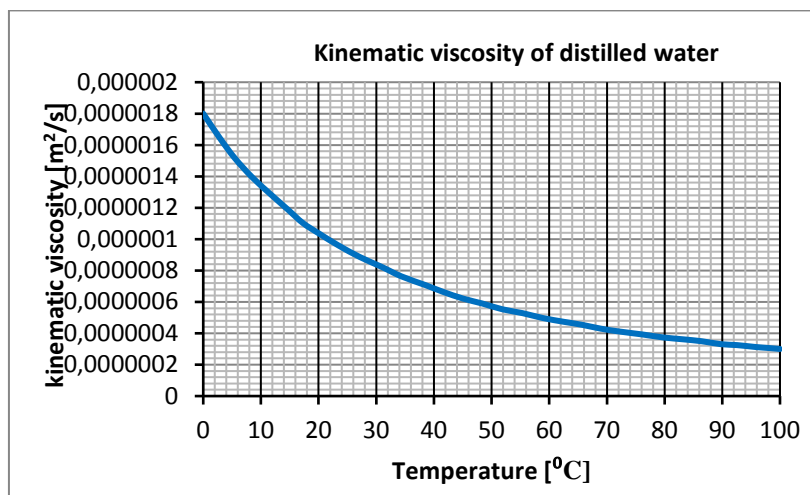
The kinematic viscosity of distilled water at a temperature of 0°C is $\nu_0 = 1.78 \cdot 10^{-6} \text{ m}^2/\text{s}$. Using the empirical relationship of variation of the kinematic viscosity of distilled water with temperature, from the manual, compute the temperature t_1 , in degrees Celsius, at which water has the kinematic viscosity $\nu_0 = 10^{-6} \text{ m}^2/\text{s}$ and the temperature t_2 , in Celsius degrees, at which water has the kinematic viscosity $\nu = 0,5 \cdot 10^{-6} \text{ m}^2/\text{s}$.

Example 11

Using the empirical equation of variation of the kinematic viscosity of distilled water with temperature, from the guide, draw, on graph paper, the graph of variation of the kinematic viscosity of distilled water for the temperature range 10°C - 25°C .

Example 12

To determine the Oswald tube constant, three measurements are made with distilled water at 20°C to establish an average flow time. The three measured times, in seconds, are $t_1 = 342.13$ s; $t_2 = 341.41$ s and $t_3 = 344.27$ s. Find the viscosity of water from the graph and compute the constant of this device.

**Example 13**

A volume of ethanol at a temperature of 20°C has the following flow times through the capillary of the Oswald viscometer, measured in seconds: $t_1 = 508$ s; $t_2 = 512$ s, $t_3 = 515$ s, $t_4 = 518$ s and $t_5 = 525$ s. Find the average

value of the dynamic viscosity of ethanol ν_e if the Oswald tube constant is known $k = 2,9112 \cdot 10^{-9} \text{ m}^2 / \text{s}^2$.

Example 14

The flow times for a volume of M30 oil at 60°C through the capillary of the Oswald viscometer, to establish an average time, are: $t_1 = 1221$ seconds; $t_2 = 1228$ seconds and $t_3 = 1223$ seconds. The flow time for the same volume of aircraft engine oil, also at 60°C, is $t_u = 2211$ seconds. Determine the kinematic viscosity of aircraft engine oil ν_u if the viscosity of M30 oil is known. 60°C, $\nu_{M30} = 1,0023 \cdot 10^{-6} \text{ m}^2 / \text{s}$.

Example 15

If a volume of ethyl alcohol flows through the capillary of the Oswald viscometer in the time $t_e = 525.6$ seconds, calculate the time t_m in which the same volume of methyl alcohol at the same temperature will flow through the same viscometer. The kinematic viscosity of ethyl alcohol, $\nu_E = 1.520 \cdot 10^{-6} \text{ m}^2 / \text{s}$, and that of methyl alcohol, $\nu_M = 0,737 \cdot 10^{-6} \text{ m}^2 / \text{s}$, are known.

Example 16

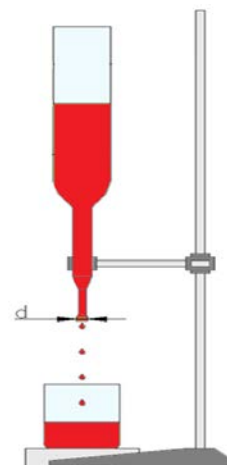
Find the average dynamic viscosity of a 10W40 engine oil at 25°C using the Brookfield rotational viscometer if only the rotating body with number 1 is used, at speed 12 for which the readings are $\alpha_1 = 50.7$; $\alpha_2 = 49.5$; $\alpha_3 =$

50.0; $\alpha_4 = 49.1$; $\alpha_5 = 48.2$. The viscometer constant, a function of speed and rotating body, is in the table below.

Speed level	Rotational bodies number			
	1	2	3	4
6	10	50	200	1000
12	5	25	100	500
30	2	10	40	200
60	1	5	20	100

Example 17

Find the average value of the dynamic viscosity of a detergent at a temperature of 25°C with the Brookfield rotational viscometer if the rotating body with number 2 is used, at speed 6 for which the reading $\alpha_1 = 26$ and at speed 12, $\alpha_2 = 53$; and with the rotating body with number 3, at speed 6 for which the reading $\alpha_3 = 6$ is taken; and at speed 12, $\alpha_4 = 12$. The viscometer constant, a function of speed and rotating body is in the table above.



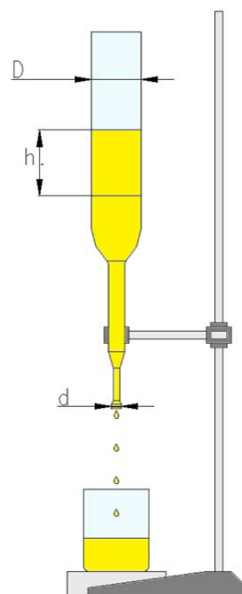
Example 18

The diameter of the capillary tube of a stalagmometer is $d = 3$ mm. It is charged with oil at a temperature of 20°C which is allowed to drip into a glass. The number of drops recorded is $n = 153$. The mass of oil dripped into the glass is determined using a scale and has the value $m = 5$ g. Knowing that the gravitational acceleration is $g = 9.81$ m/s², calculate the surface tension of the oil.

Example 19

A Traube tube has a capillary diameter $d = 2$ mm and a liquid reservoir with a diameter $D = 10$ mm and a height, between the lower and upper marks, $h = 60$ mm. The tube is charged with glycerin at a temperature of 20°C and is allowed to drip into a beaker.

When the glycerin surface reaches the upper mark, the drops are counted until the glycerin surface reaches the lower mark. A number $n = 141$ drops is determined. Knowing that the density of glycerin $\rho_g = 1.26$ kg/dm³ and that the gravitational acceleration is $g = 9.81$ m/s², calculate the surface tension of glycerin.

**Example 20**

For the relative method of determining the surface tension of a fluid, we use a standard liquid, whose surface tension we know. The Traube tube is loaded with a volume of standard liquid, distilled water, and let it drip. For distilled water, a number $n_0 = 53$ drops is obtained. The surface tension of distilled water is known $\sigma_0 = 73.5$ mN/m and its density $\rho_0 = 1$ kg/dm³, at a temperature of 20°C . The same Traube tube is loaded with the same volume of ethyl alcohol and let it drip. For ethyl alcohol, a number $n = 137$

drops is obtained. Knowing that the density of this ethyl alcohol is $\rho = 789.5 \text{ kg/m}^3$, calculate the surface tension of ethyl alcohol σ_{alcohol} .

Example 21

What is the mass of two liters of a liquid whose kinematic viscosity is $156.7 \cdot 10^{-6} \text{ m}^2/\text{s}$ and dynamic viscosity is $1,4 \cdot 10^{-1} \text{ Pa s}$.

Example 22

The viscosity in Engler degrees of a hydraulic oil is 22°E . Knowing that the density of the oil is $\rho = 0.850 \text{ g/cm}^3$, it is required to compute the kinematic viscosity ν and the dynamic viscosity η .

Example 23

What is the kinematic viscosity of a liquid, in Engler degrees, if its dynamic viscosity is $\eta = 1.53 \cdot 10^{-1} \text{ Pa s}$ and its density is $\rho = 0.900 \text{ kg/liter}$.

Example 24

A hydraulic oil, of petroleum nature, has at temperature $t_1 = 20^\circ\text{C}$ a measured kinematic viscosity $\nu_{20} = 3.36 \text{ cSt}$, and at temperature $t_2 = 80^\circ\text{C}$ a measured kinematic viscosity $\nu_{80} = 1.35 \text{ cSt}$. Using the formula for calculating the variation of kinematic viscosity with temperature, for hydraulic fluids, from the manual, calculate the kinematic viscosity of this fluid ν_{100} at a temperature of 100°C , ($1\text{cSt} = 1\text{mm}^2/\text{s} = 10^{-6}\text{m}^2/\text{s}$).

Example 25

At a temperature of 10°C , a hydraulic oil has a kinematic viscosity $\nu_{10} = 17.23 \text{ cSt}$ and at a temperature of 70°C , a kinematic viscosity $\nu_{70} = 12.35 \text{ cSt}$. Using the formula for calculating the variation of kinematic viscosity with temperature, for hydraulic fluids, from the manual, calculate the kinematic viscosity of this fluid ν_{40} at a temperature of 40°C , ($1 \text{ cSt} = 1 \text{ mm}^2/\text{s} = 10^{-6} \text{ m}^2/\text{s}$).

Using table 3.2 with the viscosity degrees, from the manual, according to ISO 3884-92, determine the viscosity grade corresponding to this fluid.

Example 26

The kinematic viscosity of a hydraulic oil at a temperature of 100°C is $\nu_{100} = 30 \text{ cSt}$ and at a temperature of 20°C is $\nu_{20} = 53 \text{ cSt}$. Using the formula for calculating the variation of kinematic viscosity with temperature for hydraulic fluids from the manual, calculate the temperature, in Celsius degrees, at which the kinematic viscosity of this oil is $\nu = 45 \text{ cSt}$ ($1 \text{ cSt} = 1 \text{ mm}^2/\text{s} = 10^{-6} \text{ m}^2/\text{s}$).

Example 27

A sealed tank, filled with water, is heated from a temperature of $t_0 = 30^{\circ}\text{C}$ to a temperature of $t = 60^{\circ}\text{C}$. Knowing that the coefficient of isobaric volumetric expansion of water is $\alpha = 1.8 \cdot 10^{-4} \text{ K}^{-1}$, and the coefficient of isothermal compressibility is $\beta = 4.19 \cdot 10^{-10} \text{ m}^2/\text{N}$, compute how much is

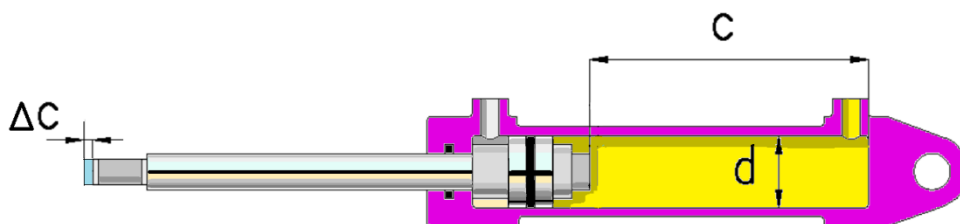
the pressure in the tank increases, if the expansion of the tank is neglected.

Example 28

Find the temperature t to which a tank filled with water, at a temperature of $t_0 = 20^\circ\text{C}$, closed hermetically, must be heated in order for the pressure inside to increase by $\Delta p = 150$ bars. Knowing that water has an isobaric volumetric expansion coefficient $\alpha = 1.8 \cdot 10^{-4} \text{ K}^{-1}$ and an isothermal compressibility coefficient $\beta = 4.19 \cdot 10^{-10} \text{ m}^2/\text{N}$. The expansion of the tank is neglected.

Example 29

In a piston cylinder as in the figure below, there is hydraulic oil that has an isobaric volumetric expansion coefficient $\alpha = 7.5 \cdot 10^{-4} \text{ K}^{-1}$. The piston has traveled a distance $c = 500$ mm. Calculate the distance Δc that the piston will travel when the oil in the cylinder expands, if the oil temperature increases by $\Delta t = 95^\circ\text{C}$. The expansion of the cylinder is neglected, and the oil is considered incompressible.

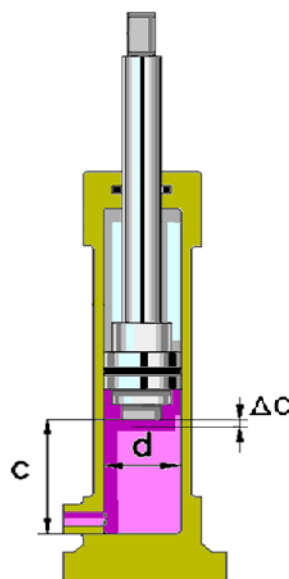


Example 30

The volume of hydraulic oil in a piston cylinder is $V = 0.5 \text{ dm}^3$. The oil has an isobaric volumetric expansion coefficient $\alpha = 8.2 \cdot 10^{-4} \text{ K}^{-1}$. The diameter of the piston is $d = 6 \text{ cm}$. What stroke Δc does the piston make if the oil temperature increases by $\Delta t = 85^\circ\text{C}$. It is assumed that the oil is incompressible and the cylinder does not expand with increasing temperature.

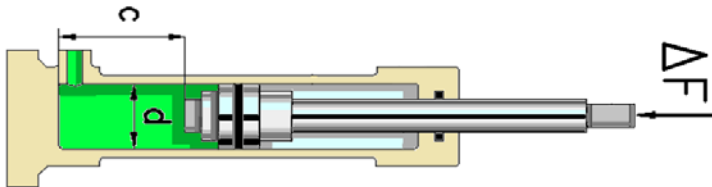
Example 31

The piston of a hydraulic cylinder is at its maximum stroke $c = 80 \text{ cm}$. The oil in the cylinder has a temperature of $t_1 = 120^\circ\text{C}$. What is the stroke Δc by which the piston retracts if the oil temperature reaches $t_2 = 10^\circ\text{C}$. The oil has an isobaric volumetric expansion coefficient $\alpha = 8.7 \cdot 10^{-4} \text{ K}^{-1}$. The oil is considered to be incompressible, and the dimensions of the cylinder do not vary with temperature.

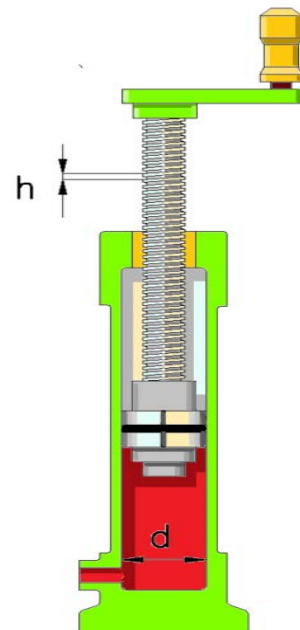


Example 32

The diameter of the piston of a hydraulic cylinder is $d = 40 \text{ mm}$. The cylinder contains hydraulic oil with the isobaric volumetric expansion coefficient $\alpha = 7.1 \cdot 10^{-4} \text{ K}^{-1}$ and the isothermal compressibility coefficient $\beta = 3.8 \cdot 10^{-10} \text{ m}^2/\text{N}$. The piston has traveled a distance of $c = 2 \text{ dm}$. Compute the variation ΔF by which the force applied to the piston rod must increase so that it does not move when the oil temperature increases by $\Delta t = 20^\circ\text{C}$. It is assumed that the cylinder does not expand with increasing temperature.

**Example 33**

A press has a cylinder piston of diameter $d = 30 \text{ mm}$ and is driven by a screw with pitch $h = 1.5 \text{ mm}$. The cylinder has a volume of $V_0 = 0.3 \text{ liters}$ of hydraulic oil with isothermal compressibility coefficient $\beta = 4.6 \cdot 10^{-10} \text{ m}^2/\text{N}$ and has the connections blocked. What is the pressure variation in the cylinder Δp if the cylinder operating lever is rotated by $n = 2$ revolutions?

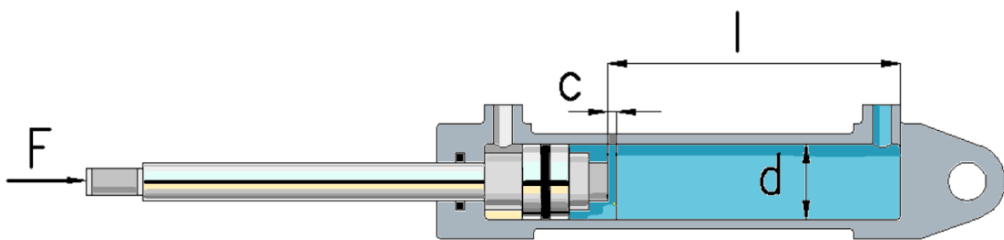


Example 34

The piston of a press is driven by a screw with a thread pitch $h = 1.7 \text{ mm}$. The press has a volume $V_0 = 0.45 \text{ liters}$ of hydraulic oil and a piston diameter $d = 30 \text{ mm}$. The hydraulic oil has an isothermal compressibility coefficient $\beta = 4.3 \cdot 10^{-10} \text{ m}^2/\text{N}$ and the connections are blocked. How many revolutions must the screw drive lever make for the pressure in the press to increase $\Delta p = 250 \text{ bar}$?

Example 35

With what force must the rod of a cylinder be pressed so that its piston moves with the stroke $c = 1 \text{ mm}$? The initial pressure in the cylinder is $p_0 = 0 \text{ bar}$. The cylinder has the connections sealed and is filled with hydraulic oil whose isothermal compressibility coefficient is $\beta = 4.6 \cdot 10^{-10} \text{ m}^2/\text{N}$. The diameter of the piston, $d = 22 \text{ mm}$, and the inner length of the cylinder, $l = 600 \text{ mm}$ are known.

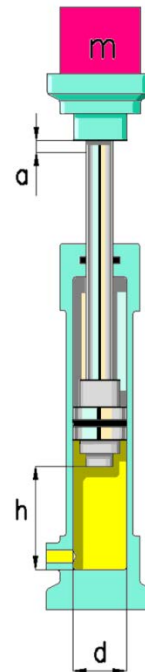
**Example 36**

A force $F = 400 \text{ daN}$ is applied to the piston rod of a cylinder, which has a closed supply connection. Knowing that the initial pressure in the cylinder

is $p_0 = 0$ bar, the piston diameter is $d = 2$ cm, and the internal length of the cylinder is $l = 600$ mm, find the stroke c that the piston will travel if the oil inside has the isothermal compressibility coefficient $\beta = 4.9 \cdot 10^{-10} \text{ m}^2/\text{N}$.

Example 37

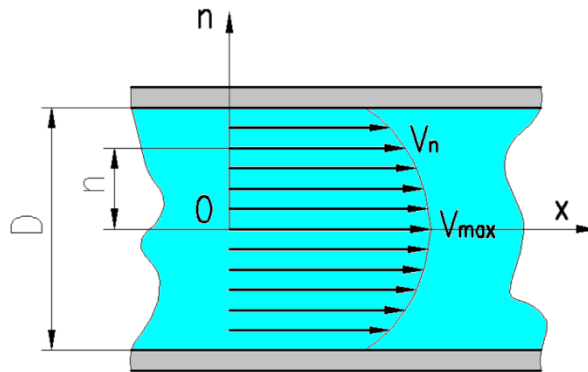
A hydraulic cylinder, used as a jack, lifts a load $m = 800$ kg, to a maximum height $h = 1$ m. Knowing that the piston diameter is $d = 4$ cm, calculate the isothermal compressibility coefficient of the hydraulic oil used, if in this position, the jack descends by $a = 1.5$ mm, when the load increases by 50%. It is assumed that there are no internal losses in the cylinder. The gravitational acceleration is $g = 9.81 \text{ m/s}^2$.



Example 38

The velocity distribution of hydraulic oil flowing through a pipe with radius $r = 25$ mm is given by the relation $v_n = v_{\max} (1 - n^2 / r^2)$, where n is the radius corresponding to the layer with velocity v_n , according to the figure below. The maximum velocity has the value $v_{\max} = 0.5 \text{ m/s}$. Determine the

minimum and maximum values of the tangential stress, if the oil has the dynamic viscosity $\eta = 9.23 \cdot 10^{-1} \text{ Pa s}$.



Example 39

Find the dynamic viscosity η and the kinematic viscosity ν of an oil with density $\rho = 850 \text{ kg / m}^3$ flowing through a pipe with diameter $d = 10 \text{ cm}$, figure above, knowing that the tangential tension at the wall is $\tau = 2.4 \text{ Pa}$, and the law of velocity variation is given by the relation $v_n = 2 - 600 n^2 \text{ m / s}$, where n is the radius corresponding to the layer with velocity v_n .

Example 40

A liquid with dynamic viscosity $\eta = 0.05 \text{ Pa}$ flows through a pipe with diameter $D = 16 \text{ mm}$. The tangential tension at the wall is $\tau = 5 \text{ Pa}$. Knowing that the law of velocity variation is given by the relation $v_n = v_{\max} (1 - 8000 n^2) \text{ m / s}$, where n is the radius corresponding to the layer with velocity v_n , determine the maximum velocity of the liquid v_{\max} in the axis of the pipe.

SOLUTIONS FOR THE PROPOSED EXAMPLES:

1. Oil density $\rho_{oil} = 880 \text{ kg/m}^3$;
2. Brake fluid column height $h_f = 103.77 \text{ mm}$;
3. Mixture density $\rho = 935.73 \text{ kg / m}^3$;
4. Mixture density in the tank $\rho = 863.88 \text{ kg / m}^3$;
5. Added oil density $\rho = 902 \text{ kg / m}^3$;
6. Additive oil density $\rho_{ao} = 866.67 \text{ kg / m}^3$;
7. Volumetric participation of liquids is:
 - liquid 1, 90.9%;
 - liquid 2, 9.1%;
8. Water density at:
 - $t = 4^\circ\text{C}$ is $\rho = 999.95 \text{ kg / m}^3$, Volume 1.00005 dm^3 ;
 - $t = 10^\circ\text{C}$ is $\rho = 999.65 \text{ kg / m}^3$, Volume 1.00005 dm^3 ;
 - $t = 20^\circ\text{C}$ is $\rho = 998.19 \text{ kg / m}^3$, Volume 1.00181 dm^3 ;
 - $t = 40^\circ\text{C}$ is $\rho = 992.25 \text{ kg / m}^3$, Volume 1.00781 dm^3 ;
 - $t = 60^\circ\text{C}$ is $\rho = 983.19 \text{ kg / m}^3$, Volume 1.01709 dm^3 ;
 - $t = 80^\circ\text{C}$ is $\rho = 971.76 \text{ kg / m}^3$, Volume 1.02905 dm^3 ;
 - $t = 100^\circ\text{C}$ is $\rho = 958.37 \text{ kg / m}^3$, Volume 1.04348 dm^3 ;
9. The kinematic viscosity of water at a temperature of 43°C is $\nu_{43} = 0.62 \cdot 10^{-6} \text{ m}^2 / \text{s}$, and at a temperature of 78°C it is $\nu_{78} = 0.35 \cdot 10^{-6} \text{ m}^2 / \text{s}$;
10. Water has a viscosity of $\nu = 0.5 \cdot 10^{-6} \text{ m}^2/\text{s}$ at a temperature of $t = 55,70^\circ\text{C}$;

11. For the graphical representation we have:

- on the Ox axis, temperature $x_1 = 10^\circ\text{C}$ and on the Oy axis, water viscosity $\gamma_1 = 1.309 \cdot 10^{-6} \text{ m}^2/\text{s}$;

- on the Ox axis, temperature $x_2 = 15^\circ\text{C}$ and on the Oy axis, water viscosity $\gamma_2 = 1.144 \cdot 10^{-6} \text{ m}^2/\text{s}$;

- on the Ox axis, temperature $x_3 = 20^\circ\text{C}$ and on the Oy axis, water viscosity $\gamma_3 = 1.010 \cdot 10^{-6} \text{ m}^2/\text{s}$;

- on the Ox axis, temperature $x_4 = 25^\circ\text{C}$ and on the Oy axis, water viscosity $\gamma_4 = 0.899 \cdot 10^{-6} \text{ m}^2/\text{s}$;

12. The constant of the viscometer is $k = 2.92 \cdot 10^{-9} \text{ m}^2/\text{s}^2$;

13. The average value of the dynamic viscosity of ethanol $\nu_e = 1.5 \cdot 10^{-6} \text{ m}^2/\text{s}$;

14. The kinematic viscosity of aircraft engine oil $\nu_o = 79.4 \cdot 10^{-6} \text{ m}^2/\text{s}$;

15. The time in which the volume of methyl alcohol flows is $t_m = 254.84$ seconds;

16. The average dynamic viscosity of 10W40 engine oil, $\eta = 247.5 \text{ mPas}$;

17. The average dynamic viscosity of detergent, $\eta = 1256.25 \text{ mPas}$;

18. The surface tension of oil is $\sigma_o = 34.01 \cdot 10^{-3} \text{ N / m}$;

19. The surface tension of glycerin is $\sigma_o = 0.0657 \text{ N / m}$;

20. The surface tension of alcohol is $\sigma_{\text{alcohol}} = 22.28 \text{ mN / m}$;

21. The mass of two liters of liquid is $m = 1.78 \text{ kg}$;

22. The viscosity of hydraulic oil is:

- kinematic is $\nu = 160 \cdot 10^{-6} \text{ m}^2 / \text{s}$;

- dynamic is $\eta = 0.136 \text{ Pas}$;

23. Kinematic viscosity, in Engler degrees, 23.26°E ;

24. The kinematic viscosity of the oil at $t = 100^\circ\text{C}$ is $\nu_{100} = 1.10 \text{ cSt}$;

25. The kinematic viscosity of the oil at $t = 40^\circ\text{C}$ is $\nu_{40} = 14.41 \text{ cSt}$ and falls into the ISO VG 15 class;

26. The oil has a viscosity of $\nu = 45 \text{ cSt}$ at the oil temperature $t = 39.98^\circ\text{C}$;

27. The water pressure increases by $\Delta p = 12887828 \text{ Pa}$;

28. The temperature at which the water in the tank reaches $t = 54.91^\circ\text{C}$;

29. The piston stroke due to oil heating is $\Delta c = 0.035 \text{ mm}$;

30. The piston stroke due to oil heating is $\Delta c = 0.012 \text{ mm}$;

31. The piston stroke due to oil cooling is $\Delta c = 0.076 \text{ mm}$;

32. The variation of the force applied to the piston is $\Delta F = 46958.5 \text{ Pa}$;

33. The variation of the pressure in the cylinder is $\Delta p = 153.66 \text{ bar}$;

34. To increase the pressure in the cylinder $\Delta p = 250 \text{ bar}$, $n = 4.025$ rotations are required;

35. To compress the oil with a stroke $c = 1 \text{ mm}$, the rod must be pressed with a force $F = 1377.29 \text{ N}$;

36. If the cylinder rod is compressed with a force $F = 400 \text{ daN}$, it moves with a stroke $c = 3.74 \text{ mm}$;

37. The isothermal compressibility coefficient of the hydraulic oil is $\beta = 4.8 \cdot 10^{-10} \text{ m}^2/\text{N}$;

38. The stresses are obtained:

- for $n = 0$ the minimum stress $\tau_{\min} = 0$ Pa is obtained;
- for $n = r = 0.025$ mm the maximum stress $\tau_{\max} = 36.92$ Pa is

obtained;

39. The dynamic viscosity is $\eta = 0.04$ Pa s, and the kinematic viscosity is $\nu = 47 \cdot 10^{-6} \text{ m}^2 / \text{s}$;

40. The maximum velocity of the liquid in the pipe axis is $v_{\max} = 0.78$ m/s;

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